A new look at fiscal sustainability: an attempt to reveal the relationship between the sustainability of foreign debt and the inequality

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Abstract

Researches on sustainability of public debt have been conducted extensively focusing on various criteria such as debt/GDP ratio, debt/export ratio, no-bubble condition, banks' home bias etc. The present paper attempts to look at the problem from a different angle. That is, following the recent literatures that pay attention to the inequality and assuming increase in share of capital income to mean increase in the inequality in a broad sense, we examine the condition for stabilizing the foreign debt. It is revealed that foreign debt converges to some value if the interest rate on the foreign debt is less than the growth rate. It is also shown that the foreign debt is sustainable if (1) share of capital income is small, (2) initial ratio of the foreign debt to GDP is small, (3) saving rate is high or (4) interest rate on the foreign debt is low.

Keywords: foreign debt, fiscal sustainability, inequality, total net worth
JEL Classification Codes: D31, E60, F34

1. Introduction

How should we evaluate the sustainability of public debt? This is the issue that has been investigated extensively with masterpieces being Bohn (1998, 2008 etc.), which focused on the ratio of debt to GDP to demonstrate that debt/GDP ratio remains constant if the primary balance is equal to zero and the growth rate is equal to the interest rate. It is also shown that if the debt/GDP ratio increases due to unexpected spending needs or low economic growth, fiscal policy must respond so as to equate the debt to the present value of primary surpluses. These Bohn's results are examined in more complex frameworks by Celasun, Debrun and Ostry (2006), Collignon (2012) etc.

For another criterion, debt/ export ratio is worth to mention. Roubini (2001) claims that the debt/exports ratio should not increase forever in order to keep the debt sustainable, while Cho

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(2015) points out that the debt/export ratio may be a faulty measure for solvency of large countries. Banks' home bias (*i.e.*, domestic banks' holdings of domestic sovereign debt) has also drawn attention. Gennaioli, Martin and Rossi (2014) show that home bias reduces the probability of default on public debt, while Asonuma, Bakhache and Hesse (2015) demonstrate that higher home bias is associated with higher debt levels.

There is another line of research that examines whether public debt satisfies no-bubble condition, *i.e.*, whether public debt series are stationary. Hamilton and Flavin (1986) and Trehan and Walsh (1991) show that the US public debt series are stationary so that the government can balance its budget in the long-run, while Kremers (1988) and Wilcox (1989) claim that the US public debt series are non-stationary. In addition to these studies, Mendoza and Oviedo (2006) derive the maximum debt level below which governments can maintain expenditures. Yakita (2008) also shows that there is a threshold level of initial public debt to keep public debt sustainable and that the threshold level is increasing in the stock of public capital. Chalk (2000), on the other hand, investigates if present value budget balance leads to the sustainability of public debt, to show that the present value budget balance may not be crucial for the sustainability of public debt.

The present paper attempts to look at the problem from a different angle. That is, following the recent literatures that pay attention to the inequality (D'Erasmo and Mendoza (2013), Azzimonti, de Francisco, and Quadrini (2014) etc.) and assuming increase in share of capital income to mean increase in the inequality in a broad sense as in Piketty (2014) etc., we examine the condition for stabilizing the foreign debt. Based on a model that extends Fujita (2015), we will reveal that foreign debt converges to some value if the interest rate on the foreign debt is less than the growth rate. It is also shown that the foreign debt is sustainable if (1) share of capital income is small, *i.e.*, inequality is small, (2) initial ratio of the foreign debt to GDP is small, (3) saving rate is high or (4) interest rate on the foreign debt is low.

2. Basic model

Let us consider a country that has a debt to foreign countries and pays it back every period by using its tax revenue. For the simplicity of analysis, we assume that domestic debt of the country is much less than its foreign debt so that the country's debt is covered mainly by foreign countries, as well as the country's foreign credit is negligible so that the foreign debt in this model is equivalent to the net foreign debt. In the following, we specify the accumulation dynamics of the foreign debt as

$$\frac{dF}{dt} = rF(t) - \tau Y(t) \tag{1}$$

where $F(t)$ and r denote the foreign debt in period t and the interest rate on the foreign debt, respectively, while $Y(t)$ is the GDP in period t and τ is a constant fraction of GDP that is allocated for repayment of the foreign debt. Here, in order to simplify the analysis, r and τ are assumed to be constant over time and the government is assumed to have no plan to issue new bonds.

With reference to the production process, normalizing the amount of labor to be unity, we assume the following AK production function

$$Y(t) = AK(t) \quad (A > 0), \tag{2}$$

where $K(t)$ stands for capital in period t, whose accumulation dynamics is

$$\frac{dK}{dt} = sY(t), \tag{3}$$

as in Harrod (1939), Piketty (2014) etc., where s is a constant fraction of GDP that is saved in every period.

Now, if we let $f(t)$ denote $F(t)/Y(t)$, the ratio of the foreign debt to GDP in period t, we have its derivative with respect to t as

$$\frac{df}{dt} = \frac{dF(t)}{dt}\frac{1}{Y(t)} - \frac{F(t)}{Y(t)}\frac{\frac{dY(t)}{dt}}{Y(t)}. \tag{4}$$

Since $\dfrac{dF}{dt} = rF(t) - \tau Y(t)$ from (1) and $F(t)/Y(t) = f(t)$ by definition, it follows that

$$\frac{df}{dt} = -\tau + (r - g)f(t), \tag{5}$$

where g is the growth rate of GDP, which turns out to be constant at sA by calculating $g(t) = (dY/dt)/Y(t)$. The growth rate g is also described as $g(t) = s/k(t)$ if we let $k(t)$ denote $K(t)/Y(t)$.

By solving differential equation (5), we have the dynamics of $f(t)$ as

$$f(t) = \frac{\tau}{r - g} + C_f e^{(r-g)t}, \tag{6}$$

where C_f is an arbitrary positive constant, which we specify as $C_f = f_0 - \dfrac{\tau}{r - g}$ by assuming the initial condition to be $f(0) = f_0$.

Condition for the foreign debt to be sustainable

In order for the foreign debt to be sustainable, it is necessary that $f(t)$ decreases and converges to some value as time goes by.

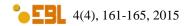We see from (6) such sustainability condition is $r<g$, to obtain the following proposition.

> *Proposition1:*
> *Foreign debt converges to some value if the interest rate on the foreign*
> *debt is less than the growth rate.*

This proposition is equivalent to say that what prevents capital to be concentrated in the future ($r<g$) also guarantees a sustainability path for debt. To put it differently, condition for an ever increasing share of capital in the economy ($r>g$), is also coincident with the debt-unsustainability condition. Since Piketty showed that $r>g$ has held for most of the past, it follows that the debt-sustainability condition has not been satisfied for the countries with foreign debt.

In the present model, since the domestic debt is negligible, $K(t)-F(t)$ represents the total net worth. Now, if we let $\beta(t)$ denote the ratio of the total net worth to GDP (*i.e.*, [K(t)–F(t)]/Y(t) $\equiv k(t) - f(t)$) and make use of

$$k(t) < \frac{s}{r}, \tag{7}$$

which is derived by combining $r<g$ with $g(t)=s/k(t)$, we have

$$\beta(t) < \frac{s}{r} - f(t), \tag{8}$$

the relationship $\beta(t)$ must satisfy to keep the foreign debt sustainable.

It is clear from the definition of $\beta(t)$ that increasing foreign debt reduces $\beta(t)$, making it easy for the inequality (8) to hold. Increase in the foreign debt, on the other hand, reduces the right hand side of (8), which makes it difficult for the inequality (8) to hold. The inequality (8) demonstrates the consequence of these two effects, *i.e.*, the condition where the foreign debt is sustainable.

As we can see from (6), if $r<g$, $f(t)$ is a monotone decreasing function with maximum point at $t=0$. Thus, we have the sufficient condition for (8) as

$$\beta(t) < \frac{s}{r} - f_0. \tag{9}$$

Assuming smaller $\beta(t)$ means smaller share of capital income as well as smaller inequality as in Piketty (2014) etc., we obtain the following proposition.

> *Proposition2:*
> *Foreign debt is sustainable if (1) share of capital income is small, i.e.*
> *inequality is small ($\beta(t)$ is small), (2) initial ratio of the foreign debt to*
> *GDP (f_0) is small, (3) saving rate (s) is high or (4) interest rate on the*
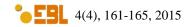> *foreign debt (r) is low.*

Let us take Greece as an example to verify the validity of this proposition. From this proposition, we can say that in order to avoid further disaster it is necessary to lower the share of capital income in Greece, to cut Greece's initial debt or to increase Greece's saving rate, which is consistent with the open letter by Thomas Piketty, Jeffrey Sachs and three others on July 7, 2015 (Piketty *et al.* (2015)). We can also say from proposition 2 that decrease in the interest rate reduces the possibility of expansion of the foreign debt.

3. Conclusion

In the present paper, we examined the condition for stabilizing the foreign debt, following the recent literatures that pay attention to the inequality and assuming increase in share of capital income to mean increase in the inequality. It was revealed that foreign debt converges to some value if the interest rate on the foreign debt is less than the growth rate. It was also shown that the foreign debt is sustainable if (1) share of capital income is small, *i.e.*, inequality is small, (2) initial ratio of the foreign debt to GDP is small, (3) saving rate is high or (4) interest rate on the foreign debt is low.

In the aftermath the recent financial crisis in Europe, many countries have taken austerity measures to reduce the debt levels, which have brought about a considerable degree of controversy (Piketty *et al.* (2015) etc.). It is our hope that this study will contribute to the progress of this field. For that purpose, it is necessary to examine the robustness of the results by assuming more general production function. It is also necessary to incorporate the domestic debt as well as the foreign credit. We take up such analysis next.

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