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General equivalency between the discount rate and the going-in and going-out capitalization rates

Gaetano Lisi*

Department of Economics and Law, University of Cassino and Southern Lazio, Italy

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Abstract

This theoretical paper extends the useful work of Sevelka (2004) concerning the equivalence between discount rate and capitalization rate. The extension concerns two main points. First, it frames this relation within a clear theoretical framework, that starts from the net present value and goes on to the direct capitalization, passing through the yield capitalization. Second, it also provides an equivalence relation between discount rate and going-in and going-out capitalization rates. These equivalence relations turn out to be crucial for real estate appraisals.

Keywords: real estate appraisal; yield capitalization; direct capitalization; discount rate; going-in capitalization rate; going-out capitalization rate
JEL Classification Codes: L85, R21, R31

1. Introduction

This paper extends the work of Sevelka (2004) concerning the equivalence between discount rate and capitalization rate in two ways. First, it frames the relation between discount rate and capitalization rate inside two well-known models of economics and finance (investment decisions and yield capitalization). In this way, the theoretical foundations of the capitalization rate are very clear and, at the same time, it is possible to obtain a straightforward and quick review on all the related topics. Second, this paper provides a general equivalence relation between discount rate and capitalization rate, thus also including the so-called going-out capitalization rate. Indeed, the important distinction between the going-in and going-out capitalization rates

* E-mail: gaetano.lisi@unicas.it.

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is often disregarded or not fully revealed. This equivalence relation is, instead, crucial for estimating the resale value of the property, and thus the market value of the house, since the reversion value represents the larger share of the total return of a property investment. From a theoretical point of view, the formula of the going-out capitalization rate is merely an updating – at the end of the holding period of the property – of the going-in capitalization rate formula. Nevertheless, from an empirical point of view, it is far from trivial, since the expectations about the key macroeconomic variables (rates of interest, growth and inflation) play a key role.

This paper is organised in two sections: Section 2 provides a clear and quick review on the theoretical foundations of the capitalization rate; whereas, Section 3 includes the going-out capitalization rate into the equivalence relation between discount rate and capitalization rate, thus getting a general equivalence relation.

2. Theoretical foundations of the capitalization rate

In real estate economics there is a very close link (although it is not always highlighted) between the investment decisions – carried out through the *net present value* (*NPV*) *approach* – and the property valuations, performed with financial and income methods, such as the *discounted cash flow* (*DCF*) analysis and the *yield capitalization* method[1]. It is well-known that the financial and income methods should be used as valuation models when real estate is capable of generating rental income and an investor represents the most likely purchaser (Sevelka, 2004). Actually, these two conditions are often fulfilled, since each house has a (potential) rental value and households behave like investors in some cases; for example, when for family or work reasons, they need to buy a new house and, at the same time, they try to sell or rent their home ownership.

The *net present value* (*NPV*) *approach* is used to determine the advisability of investing. Precisely, the investment is profitable, and thus it should be undertaken if the *NPV* is positive or at least non-negative, viz.:

$$NPV \equiv \sum_{t=1}^{n}\left(\frac{R_t}{(1+r)^t}\right) - P \geq 0 \tag{1}$$

where t is the time and n is the investment period. In words, the *NPV* compares the cost of investment P (that is sustained today) with its future revenues R_t, discounted up to the present time by an appropriate or risk-adjusted discount rate r[2]. In the case where all the benefits are received today, namely $n = 1$, equation (1) becomes a simple comparison between the rate of return of the investment (R - P) / R and the market interest rate (the cost for financing or the opportunity cost):

$$\frac{R-P}{P} \geq r \xrightarrow{yields} \frac{R}{(1+r)} - P \geq 0.$$

[1] In Sevelka (2004), the terms yield capitalization method and discounted cash flow (DCF) analysis are used interchangeably. For the sake of simplicity, we follow this approach.

[2] The discount rate is the rate of return on capital which considers all future expected benefits, including the revenue from sales at the end of the holding period (Appraisal Institute, 2002). The appropriate or risk-adjusted discount rate is usually divided into two major components (see, for example, Corgel, 2003; Clayton and Glass, 2009), namely $r = rf + RP$, where rf is the free-risk rate and RP is the risk premium associated with the real estate investment.

The *NPV* formula, therefore, generalises this simple comparison to the (very realistic) case where $n > 1$.

The *yield capitalization* or the *income approach* is used to determine the value of an asset, starting from the benefits (income stream) that it is able to generate in the course of time t. Evidently, also in this case one must consider the future income streams, discounted up to the present time. Indeed, the equation of *yield capitalization* can be obtained from the *NPV* formula, i.e. equation (1), under the indifference condition between investing or not investing, i.e. $NPV = 0$. By using the *Net Operating Income* (*NOI*), in place of a generic future stream of income, in fact, we get from (1) the standard formula of the *yield capitalization*:

$$\sum_{t=1}^{n}\left(\frac{NOI_t}{(1+r)^t}\right) = P \tag{2}$$

where P is the house price or its market value[3], $NOI_t \equiv R_t - C_t$ is the *Net Operating Income*, R_t is the gross rent, C_t are the financing and operating costs, r is the purchaser's opportunity cost, namely the discount rate, and n is the property's useful life. The economic meaning of equation (2) is straightforward: in equilibrium, the price of a property should equal the present value of its expected benefits (net rental income)[4]. Equation (2) can be decomposed into three parts, in order to highlight the key components of the house value (see Phillips, 1988):

$$P = \frac{NOI_1}{1+r} + \sum_{t=2}^{k}\frac{NOI_t}{(1+r)^t} + \sum_{t=k+1}^{n}\frac{NOI_t}{(1+r)^t} \tag{2'}$$

where $\frac{NOI_1}{1+r}$ is the discounted NOI at the end of the first period; $\sum_{t=2}^{k}\frac{NOI_t}{(1+r)^t}$ is the sum of the discounted NOI during the property holding period k, and $\sum_{t=k+1}^{n}\frac{NOI_t}{(1+r)^t}$ are the proceeds from the sale, namely the present value of the net rental flow for the property's remaining useful life. Equation (2'), therefore, distinguishes within the economic life of the property n, the property holding period (until time k) and the remaining useful life of the property (after time k).

By using the two usual hypotheses of the well-known *Gordon model*, namely a steady increase of the net operating income at rate g and an economic life of the property that is very long (mathematically, $n \to \infty$), equation (2') becomes:

$$P = \frac{NOI_1}{1+r} + \frac{NOI_1\cdot(1+g)}{(1+r)^2} + \frac{NOI_1\cdot(1+g)^2}{(1+r)^3} + \cdots = NOI_1 \cdot \left[\frac{1}{1+r} + \frac{(1+g)}{(1+r)^2} + \frac{(1+g)^2}{(1+r)^3} + \cdots\right] = NOI_1 \cdot \sum_{t=0}^{\infty}\frac{(1+g)^t}{(1+r)^{t+1}} \tag{2''}$$

The second hypothesis (a very long economic life of the property) is not unrealistic for real estate. Note that equation (2'') emphasises the role of the net operating income of the first period, NOI_1. Indeed, if the condition $|g| < r$ is fulfilled (namely the house price has a positive and finite value) then the geometric series converges, i.e. $\sum_{t=0}^{\infty}\frac{(1+g)^t}{(1+r)^{t+1}} = \frac{1}{r-g}$, and thus equation (2'') collapses to:

[3] *"Market value is a concept in economic theory and cannot be observed directly. Sales prices provide the most objective estimates of market values and under normal circumstances should provide good indicators of market value."* (IAAO, 2013, p. 7).

[4] In effect, as regards the variable R_t, Phillips (1988) talks about "market-clearing rents".

$$P = NOI_1 \cdot \left[\frac{1}{r-g}\right] \tag{3}$$

Equation (3) represents the *direct capitalization method*, from which it is a straightforward procedure to obtain the capitalization rate (c):

$$\underbrace{c = (r-g)} = \frac{NOI_1}{P} \tag{4}$$

that is, the capitalization rate is equal to the discount rate less the growth rate of the net operating income. Equation (3) tells us that one can obtain an estimate of the property value by applying the capitalization rate to the net operating income of the first period. It is quite clear that *yield capitalization* and *direct capitalization* are interrelated valuation models and thus applying either approach to the same income-producing property should generate a similar estimate of the market value (Etter, 1994; Sevelka, 2004). Intuitively, one should get the same estimate under the two hypotheses previously analysed.

Without the two simplifying assumptions first introduced (a steady growth in income and an infinite economic life of the property), the capitalization rate is approximately equal to the discount rate less the growth rate of the net operating income:

$$c \approx r - g \tag{4'}$$

equation (4') is the equivalence relation used by Sevelka (2004) and represents an approximation of the close relationship between discount rate and capitalization rate in equation (4). Indeed, Sevelka (2004) distinguishes between real growth and nominal or inflationary growth:

$$c \approx r - g - \pi \tag{5}$$

thus obtaining an equivalence relation between "real" discount rate and capitalization rate:

$$c \approx \rho - g \tag{5'}$$

where $\rho = (r - \pi)$ is the real discount rate, π is the (expected) inflation rate and g is the real income growth rate.

3. General equivalency between discount rate and going-in and going-out capitalization rates

In real estate appraisal, equation (2') often involves an independent and separate estimate of the third term, namely the proceeds from the sale $\sum_{t=k+1}^{n} \frac{NOI_t}{(1+r)^t}$. In this case, the formula of the *yield capitalization* is the following:

$$P = \frac{NOI_1}{1+r} + \sum_{t=2}^{k} \frac{NOI_t}{(1+r)^t} + \frac{V_R}{(1+r)^k} \tag{6}$$

where V_R is the so-called *exit value*, *scrap value* or *going-out value*. Concisely, equation (6) tells us that the proceeds from the sale are no longer calculated as the present value of future flows of net rental income, subsequent to the holding period k, but instead, the calculation of the resale value implies an independent and separate assessment of the property value at the end of the holding period, time k. The estimation of the reversion or resale value is of crucial importance, since it represents the larger share of the total return of a property investment (Appraisal Institute, 2001). Of course, in equation (6), V_R is discounted to obtain its value at the present time.

By using direct capitalization, the estimate of the (potential) resale value is given by:

$$V_R = \frac{NOI_{k+1}}{cf} \tag{7}$$

where NOI_{k+1} is the net operating income after the holding period k and cf is the capitalization rate. Highly intuitively, the rate that capitalises the net operating income of the first year in the house value in the direct capitalization – namely, the rate c in equation (3) – cannot be used, in general, as the rate that capitalises the net operating income after the holding period in the resale value of the property – namely, the rate cf in equation (7). Indeed, in the real estate literature, the rate c is known as the *"going-in capitalization rate"*; whereas, the rate cf is known as the *"going-out capitalization rate"* (Wang *et al.*, 1990; Sevelka, 2004).

In an influential work, Wang *et al.* (1990) provide a rigorous mathematical approach to derive the appropriate capitalization rate in order to estimate the expected resale price or reversion value, i.e. the "going-out capitalization rate", starting from the "going-in capitalization rate". According to Wang *et al*. (1990), there is no fixed relationship between the two capitalization rates, since the going-out capitalization rate may be greater, equal or lower than the going-in capitalization rate. Precisely, "[…] *the going-in and the going-out capitalization rates should be the same if there is no reason to assume that income growth rates, required rates of return, or property appreciation rates are different during and after the projected holding period.*" (Wang *et al.*, 1990, p. 235).

In the present paper, we suggest a similar but simpler way to estimate the going-out capitalization rate that does not require the knowledge of the going-in capitalization rate. Practically, we translate the key insight of Sevelka (2004, p. 140) into a formula, whereby, "*the going-out capitalization rate can be also seen as the next purchaser's going-in capitalization rate*". In brief, the estimate of the going-out capitalization rate is merely an "updating" of the going-in capitalization rate at the end of the holding period, when an independent and separate assessment for estimating the expected resale price or reversion value is carried out. The going-out capitalization rate can thus be approximated in the following way. Firstly, it needs to introduce the time element (denoted by the subscript t) into the equivalence relation between "real" discount rate and capitalization rate, namely equation (5'):

$$c_t \approx \rho_t - g_t \tag{8}$$

Of course, at the beginning of the holding period, i.e. for $t = 1$, the previous equation refers to the *going-in capitalization rate*:

$$c \equiv c_1 \approx \rho_1 - g_1 \tag{9}$$

Eventually, it is possible to obtain *the going-out capitalization rate* simply by evaluating the previous equations for $t = k$, i.e. at the end of the holding period:

$$cf \equiv c_k \approx \rho_k - g_k \tag{10}$$

Intuitively, time k is the beginning of the holding period for the next buyer. Indeed, the going-out capitalization rate "becomes" the going-in capitalization rate of the next buyer. Eventually, three main results emerge from this analysis:

(1) The equivalency between discount rate and going-out capitalization rate follows the equivalency between discount rate and going-in capitalization rate. Indeed, as shown above, the relation between cf and ρ in equation (10) is the same as the relation between c and ρ in equation (9). Hence, there is a general and unique equivalency between real discount rate and capitalization rate and is given by equation (8), where, however, the time factor plays a key role.

(2) The *going-out capitalization rate* (cf) can be higher, lower or equal to the *going-in capitalization rate* (c), viz.:

$$(cf - c) \approx (\rho_k - \rho_1) - (g_k - g_1) \tag{11}$$

Precisely, the *going-out capitalization rate* is higher than the *going-in capitalization rate*, namely $cf > c$, if the nominal discount rate at the end of the holding period is higher than the nominal discount rate at the beginning of the holding period, and/or the real income growth rate at the end of the holding period is lower than the real income growth rate at the beginning of the holding period, and/or the inflation rate at the end of the holding period is lower than the inflation rate at the beginning of the holding period. Instead, the *going-out capitalization rate* is lower than the *going-in capitalization rate*, i.e. $cf < c$, if the nominal discount rate at the end of the holding period is lower than the nominal discount rate at the beginning of the holding period, and/or the real income growth rate at the end of the holding period is higher than the real income growth rate at the beginning of the holding period, and/or the inflation rate at the end of the holding period is higher than the inflation rate at the beginning of the holding period. Obviously, the going-out capitalization rate is equal to the going-in capitalization rate, if there are no change in the rates of return on capital, growth and inflation between the beginning and the end of the holding period.

(3) As an important component of equation (6), the estimate of the resale value (V_R) is aimed at the estimation of the house market value and, thus, it is carried out at the present time[5]. Hence, the going-out capitalization rate cf in equation (10) turns out to be an expected rate:

$$cf \approx \rho_k^e - g_k^e \tag{10'}$$

where the superscript "e" denotes the expectations about the rates. The equivalence relation between going-in capitalization rate and going-out capitalization rate, therefore, depends on the expected rates of return on capital, growth and inflation. It follows that the expectations about the trend over time of the key macroeconomic variables also play the most important role in the real estate appraisals.

Basically, the awareness of these equivalence relations is crucial in performing proper real estate appraisals. In the estimation process, in fact, the evaluator relies (still and almost exclusively) on his experience and expertise. This is why the "estimate" is still considered to be an art more than a science (Lentz and Wang, 1998; Lai *et al.*, 2008).

[5] Intuitively, the estimate of the resale value at the end of the holding period equates to the estimate of the house price.

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