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# Alternative specifications of human capital in production functions

**Luis Orea • Antonio Alvarez**[*]

*Department of Economics, University of Oviedo, Spain*

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**Abstract**

In this paper we argue that some empirical specifications may not be appropriate to estimate accurately the returns to human capital. In particular, we show that the inclusion in aggregate production functions of the ratio of skilled labor force over total labor force as a proxy for human capital may not be a good way to control for the heterogeneity of labor.

*Keywords*: Human capital; Production function; Specification
*JEL Classification Codes*: E24, I25, J24, O40

## 1. Introduction

The concept of human capital has become the focus of attention by economists since the seminal works by Mincer (1958), Schultz (1960, 1961) and Becker (1962). As a result, there is wide consensus among the economic profession about the importance of human capital as a source of productivity improvements. In fact, the endogenous growth theory assigns to human capital a key role in the explanation of economic growth (Romer, 1990; Howitt, 2010; Day and Dowrick, 2013).

However, some empirical work has failed to find positive or significant estimates of the productivity of human capital investment. For example, Kyriacou (1991) found the coefficient of human capital negative although non-significant. Similar results were obtained by Benhabib and Spiegel (1994). Pritchett (1996) also could not find a positive contribution of human capital to aggregate economic growth.

This unexpected result has been a source of concern for many researchers. For example, Pritchett (1996) states: "The claim that expanding education is good for economic growth seems intuitively obvious, receives apparent empirical support from both individual and aggregate data, and has become a fundamental tenet of development strategy. However, like many beliefs the empirical basis for this claim is substantially weaker than is often supposed". In a similar vein, Judson (2002) indicates that "Despite the conventional wisdom that output growth and human capital should be positively correlated, statistically significant results have been mixed, and strong and positive correlations between growth and human capital accumulation have been the exception rather than the rule".

[*] Corresponding author. E-mail: alvarez@uniovi.es.

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Although there is no consensus among economists about the possible reasons for these counterintuitive findings, several explanations have been put forward. Temple (1999) argued that the problem lies in the unobservable country heterogeneity. By excluding some influential observations from the Benhabib and Spiegel (1994) dataset, he finds that human capital is significant. Mulligan and Sala-i-Martín (2000) argue that using average years of schooling as a proxy for human capital to explain economic growth may be misleading. They build up a set of human capital indexes for the United States and find that human capital grew twice as fast as average year of schooling. Hanushek and Kimko (2000) and Hanushek and Woessman (2012) address the measurement of labor-force quality by constructing "new measures of quality based on student cognitive performance on various international tests of academic achievement in mathematics and science", raising the explanatory power of growth models. Other papers have argued that the cause can be due to poor data quality (de la Fuente and Domenech, 2001; 2006). Wöβman (2003) does a comprehensive review of most of the measures of human capital employed in the empirical literature.

Despite the strong emphasis on measurement issues around how to proxy human capital, the above-mentioned problem of human capital not being significant in many regression models can be due to other factor. As Hanushek and Kimko (2000) indicate: "Two issues arise in considering the effect of human capital on economic growth: how should any relationship be specified and how should human capital be measured" In this paper we pay attention to the first issue, which has received less attention. We argue that some empirical specifications may not be appropriate to estimate accurately the returns to human capital.[1] In this sense, Nelson and Phelps (1966) point out that "…the usual straightforward insertion of some index of educational attainment in the production function may constitute a gross misspecification of the relation between education and the dynamics of production". Specifically, we argue that the inclusion in aggregate production functions of the ratio of skilled labor force over total labor force as a proxy for human capital may not be a good way to control for the heterogeneity of labor.

We compare two alternative specifications to account for the heterogeneity in the labor force (workers with different levels of education). We show that both specifications imply very different characteristics of the underlying technology. In particular, these differences are important to assess the returns to education. We carry our analysis in the context of a Cobb-Douglas production function. We think that the paper shows the importance of specification and can serve as a warning to practitioners that some measures of human capital, even if they may sound sensible, can impose undesirable restrictions on the technology being estimated.

The structure of the paper is the following. In the next section we develop the two alternative specifications. In section 3 we compare them. In section 4 we provide some empirical evidence. In Section 5 we present some conclusions.

## 2. Human capital in a Cobb-Douglas production function

The aggregate production function can be written in general terms as:

$$Y = F(K, L) \tag{1}$$

where Y is aggregate output, K is physical capital, and L is total labor. In this formulation, changes in human capital or labor quality are not controlled for. The natural way to account for labor quality is to explicitly allow for different types of labor. This implies a production function such as:

$$Y = F^{S}(K, L_1, L_2) \tag{2}$$

[1] It must be noted that other papers have paid attention to the importance of the specification of human capital in aggregate production functions. One example is the debate about the specification of human capital in levels or in rates (e.g., Kyriacou, 1991).

where $L_1$ and $L_2$ stand for skilled and non-skilled workers respectively and $L = L_1+L_2$. We will refer to this function as the 'Standard Production Function' (therefore, superscript S). This type of specification is rather common in the empirical literature that studies the substitution of skilled for unskilled labor (e.g., Mello, 2008)

However, the production function has been most of the times estimated replacing both types of labor ($L_1$ and $L_2$) by aggregate labor (L) and a human capital variable (H).

$$Y = F^M(K, L, H) \tag{3}$$

The usual way to proxy human capital has been the years of schooling of the labor force (e.g., Barro, 2001), the enrollment rates (e.g., Mankiw et al., 1992), or some variants of both. For example, Judson (2002) modifies the well-known Barro-Lee database (Barro and Lee, 1993) on educational attainment to create a measure of human capital in value terms by using data on expenditure on education to weight the labor force.[2]

However, this index is sometimes defined as the ratio of skilled labor to total labor, $H = L_1/L$. That is,

$$Y = F^M(K, L_1 + L_2, \frac{L_1}{L_1 + L_2}) \tag{4}$$

We acknowledge that there are not many papers where this proxy has been used. Some examples are Cheng and Hsu (1997) who measure human capital as the ratio of the number of college graduates to total labor force and Freire-Serén (2002) who measures human capital as the "fraction of the employed population with at least secondary school education" although she doesn't include human capital in a production function in the way we do. However, as stated in the Introduction, our purpose is to show the importance of specification issues, regardless of how frequently they have appeared in the empirical literature.

Both production functions, $F^S(\cdot)$ and $F^M(\cdot)$, model the relationship between aggregate output and some inputs (physical capital and two types of labor). However, the replacement of the two labor inputs ($F^S(\cdot)$) by an aggregate input and an input-mix ratio that takes into account the composition of the aggregate input ($F^M(\cdot)$) may yield a different representation of the technology. This is demonstrated below for the Cobb-Douglas production function.

The Cobb-Douglas form (Cobb and Douglas, 1928) is one of the most widely used production functions. Its popularity relies on its well-known and easy-to-understand properties, as well as the fact that it is linear in parameters after taking logs. The Cobb-Douglas production function defined in terms of standard inputs (Eq. 2) can be written as:

$$Y = A \cdot K^\alpha \cdot L_1^{\beta_1} \cdot L_2^{\beta_2} \tag{5}$$

where A, $\alpha$, $\beta_1$ and $\beta_2$ are parameters. We will refer to this function as the 'standard' Cobb-Douglas (SCD).

On the other hand, a Cobb-Douglas production function with the human capital index can be written as:

$$Y = A \cdot K^\alpha \cdot L^\beta \cdot H^\gamma \tag{6}$$

where A, $\alpha$, $\beta$ and $\gamma$ are parameters to be estimated. Since $L_1$ represents skilled labor, it is expected that $\gamma>0$.

Rewriting (6) in terms of $L_1$ and $L_2$, we get:

$$Y = A \cdot K^\alpha \cdot (L_1 + L_2)^{\beta-\gamma} \cdot L_1^\gamma \tag{7}$$

We will refer to the function in (7) as the 'modified' Cobb-Douglas (MCD).

Note that the SCD and MCD specifications are not equivalent. In fact, there are no parametric restrictions that allow us to go from one specification to the other, i.e., they are not nested.

[2] Van Leeuwen and Köldvari (2008) use this measure in a time series analysis of economic growth in three Asian countries finding that there is a long-run relationship between human capital and economic growth.

Both specifications are not equivalent even if the elasticities of both types of labor are equal. In this case, $\beta_1=\beta_2$, and equation (5) becomes:

$$Y = A \cdot K^{\alpha} \cdot (L_1 \cdot L_2)^{\beta_1} \tag{8}$$

which is not equivalent to (7). However, the two specifications are equivalent when the proportion of skilled labor is the same as the proportion of non-skilled labor, i.e., $L_1=L_2$. In this case, the two functions can be written as:

$$\begin{aligned} SCD: &\quad Y = A \cdot K^{\alpha} \cdot L_1{}^{\beta_1} \cdot L_1{}^{\beta_2} = A \cdot K^{\alpha} \cdot L_1{}^{\beta_1+\beta_2} \\ MCD: &\quad Y = A \cdot K^{\alpha} \cdot (2L_1)^{\beta-\gamma} \cdot L_1^{\gamma} = A' \cdot K^{\alpha} \cdot (L_1)^{\beta} \end{aligned} \tag{9}$$

where $A'=A\cdot 2^{\beta-\gamma}$. It is clear that only in this case the two specifications are equivalent. This indicates that the key element that differentiates both functions is the relative proportion of the two types of labor, $L_1/L_2$.

## 3. Comparing the two models

We now proceed to compare these two specifications. In order to do so, in Table 1 we show the economic characteristics implied by both production functions. The properties of the SCD production function are well-known: a) marginal productivities are always positive and decreasing if the parameters in (5) are positive and smaller than one; b) output elasticities and the scale elasticity (ε) are constant; and c) the elasticity of substitution between inputs is equal to one.

*Table 1.* Production characteristics of the Standard and Modified Cobb-Douglas.

| | **Standard CD** | **Modified CD** |
|---|---|---|
| $MPK$ | $\frac{\alpha Y}{K}$ | |
| $MPL_1$ | $\frac{\beta_1 Y}{L_1}$ | $\left[\frac{\beta-\gamma}{L}+\frac{\gamma}{L_1}\right]Y$ |
| $MPL_2$ | $\frac{\beta_2 Y}{L_2}$ | $(\beta-\gamma)\frac{Y}{L}$ |
| $MRTS_{L_1L_2}=\frac{MPL_1}{MPL_2}$ | $\frac{\beta_1}{\beta_2}\frac{L_2}{L_1}$ | $1+\frac{\gamma}{\beta-\gamma}\cdot\frac{L}{L_1}$ |
| $\frac{\partial\, ln(L_2/L_1)}{\partial MRTS_{L_1L_2}}$ | 1 | $\frac{\gamma L_2+\beta L_1}{\gamma L_2}$ |
| $\varepsilon_K=\frac{\partial\, ln\, Y}{\partial\, ln\, K}$ | $\alpha$ | $\alpha$ |
| $\varepsilon_{L_1}=\frac{\partial\, ln\, Y}{\partial\, ln\, L_1}$ | $\beta_1$ | $(\beta-\gamma)\frac{L_1}{L}+\gamma$ |
| $\varepsilon_{L_2}=\frac{\partial\, ln\, Y}{\partial\, ln\, L_2}$ | $\beta_2$ | $(\beta-\gamma)\frac{L_2}{L}$ |
| $\varepsilon_L=\left.\frac{\partial\, ln\, Y}{\partial\, ln\, L}\right\rvert_{\bar{H}}=\varepsilon_{L_1}+\varepsilon_{L_2}$ | $\beta_1+\beta_2$ | $\beta$ |
| $\varepsilon=\varepsilon_K+\varepsilon_L$ | $\alpha+\beta_1+\beta_2$ | $\alpha+\beta$ |
| $\varepsilon_H=\left.\frac{\partial\, ln\, Y}{\partial\, ln\, H}\right\rvert_{\bar{L}}$ | $\beta_1-\beta_2\frac{L_1}{L_2}$ | $\gamma$ |

A quick glance at Table 1 suggests that when the human capital index is included in a Cobb-Douglas production function, the underlying technology that we are modeling has different characteristics from the technology represented by an SCD. Moreover, the economic implications that can be inferred from both production functions can vary drastically, as shown below.

a) *Marginal Productivities*: In the MCD the marginal products of both types of labor ($MPL_1$ and $MPL_2$) are positive (if $\beta$-$\gamma$>0) and decreasing, as in the SCD. However, if $\gamma$ is positive $MPL_1$ is always higher than $MPL_2$. This means that the MCD imposes a technology where the marginal productivity of the first non-skilled worker is less than the marginal productivity of an skilled worker, despite the number of skilled workers.

b) *Elasticity of Substitution*: The elasticity shown in Table 1 is the well-known Allen elasticity (that measures the percentage change in the ratio of two inputs due to a 1% change in the ratio of their prices) when inputs are paid according to their relative marginal products. Notice that, while $\partial(L_2/L_1)/\partial MRTS$ is always equal to unity in the SCD, it increases (decreases) with $L_1$ ($L_2$) in the MCD. Therefore, the Allen elasticity of input substitution between both types of labor is also different in both models. This implies that while the substitution of skilled and non-skilled labor does not depend on total labor structure, it increases in a country whose ratio of skilled to non-skilled labor increases over time.

c) *Output elasticities*: In the SCD the elasticity of output with respect to both types of labor is constant. However, in the MCD they change with the ratio of skilled to non-skilled labor. When this ratio rises, the elasticity of skilled labor increases while the elasticity of non-skilled labor decreases in the same amount, the result being that the sum of labor elasticities remains constant. This result has important implications when we use the estimated production function to predict the evolution of the income share of both types of labor. In particular, if output and input markets are competitive, skilled and non-skilled labor are in equilibrium, paid in accordance to their value marginal product, that is:

$$\begin{aligned} MPL_1 &= W_1/P \\ MPL_2 &= W_2/P \end{aligned} \tag{10}$$

where $W_1/P$ and $W_2/P$ are respectively the wage paid to skilled and non-skilled labor, normalized by the output price, $P$. Multiplying each equilibrium equation by the inverse of the corresponding average productivity of labor yields the known result that the output elasticity with respect to each type of labor is its income share. Hence, the ratio between the two labor elasticities can be interpreted as the relative gains of each type of workers in total income:

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{MPL_1 \cdot \frac{L_1}{Y}}{MPL_2 \cdot \frac{L_2}{Y}} = \frac{\frac{W_1 \cdot L_1}{P \cdot Y}}{\frac{W_2 \cdot L_2}{P \cdot Y}} = \frac{W_1 \cdot L_1}{W_2 \cdot L_2} \tag{11}$$

As shown in Figure 1, both types of productions functions predict quite different evolutions of the income share of skilled and non-skilled workers. While in the SCD neither group of workers increases its participation in total income (Figure 1a), in the MCD skilled labor increases its share when the ratio of skilled to non-skilled labor increases (Figure 1b).

There are also differences between both models regarding the output elasticity with respect to the input-mix variable (H), holding total labor constant. These differences are illustrated in Figure 2. In the SCD, an increase in the ratio of skilled to non-skilled labor has a negative effect on the elasticity, while in the MCD has no effect, regardless the proportion of skilled workers. This result may have important implications when the labor-quality ratio is changing in a country. While the elasticity in the MCD seems to indicate that promoting increases in the labor-quality ratio is always a positive policy measure, the elasticity in the SCD seems to indicate that this policy will not have the same effect in all countries. Moreover, its implementation might also have a negative effect in countries that already have a large proportion of skill workers.

*Figure 1a.* Ratio of income shares in the SCD.

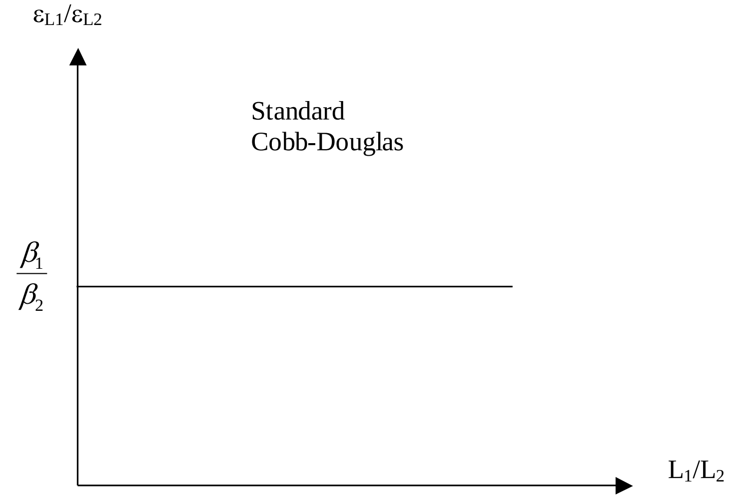



*Figure 1b.* Ratio of income shares in the MCD.

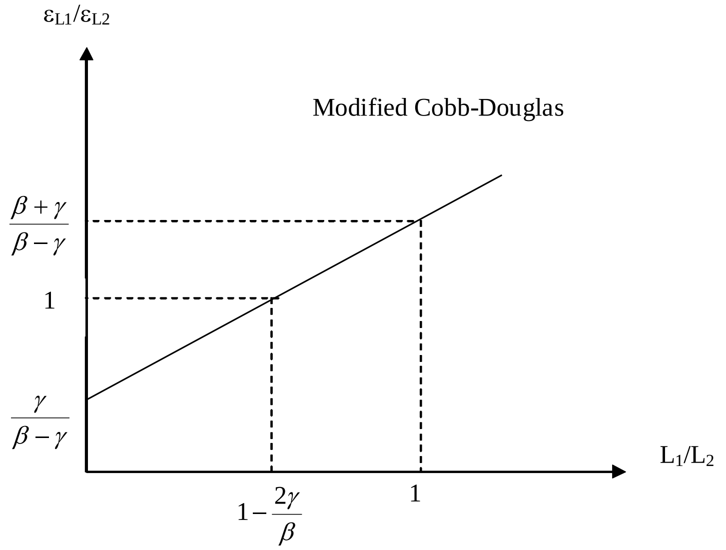
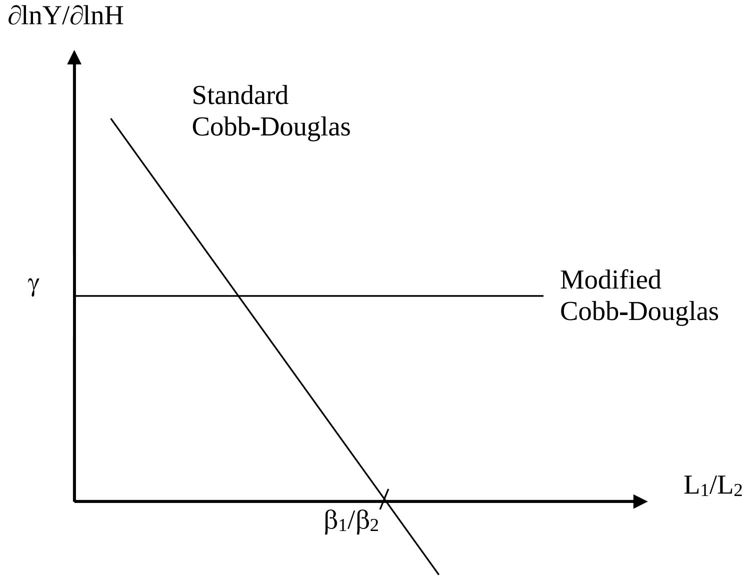



*Figure 2.* Output -elasticity with respect to the input-mix index.



d) *Scale Elasticity*: In both models the scale elasticity is constant, but the underlying reasons are different. The scale elasticity in the SCD is constant because output elasticities are always constant in this model, whereas in the MCD it is due to both output elasticities compensating each other.

Therefore, it is clear that both specifications yield different representations of the underlying technology. Which is the best one? It is hard to tell on theoretical grounds. On the one hand, the SCD is more flexible than the MCD in the representation of marginal productivities and output elasticities, while the MCD allows for more flexible substitution between skilled and non-skilled labor.

## 4. Conclusions

In the present paper we have shown that, in general, the inclusion of input-mix indices in a production function in order to capture differences in input quality or input composition may change the characteristics of the underlying technology. In particular, the conventional properties of the well-known Cobb-Douglas production function vary substantially when a human capital index is included. We show that the predictions from a Cobb-Douglas with a human capital index are, in some cases, difficult to justify from a theoretical standpoint.

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