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A new time-varying method for club convergence analysis

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Abstract

One of the most frequently used methods to test the club convergence hypothesis is the Phillips-Sul approach. However, this method does not allow the verification of whether the number, size, or structure of convergence clubs vary over time. Therefore, this study proposes a new time-varying method for identifying convergence clubs by modifying the Phillips-Sul approach. The new method was applied to data on gross domestic product per capita in selected European countries for 1995–2021. The results reveal that the number of convergence clubs, their size, and their internal structure changed over time.

Keywords: convergence club, time-varying framework, Phillips-Sul approach, local linear log *t* regression
JEL Classification Codes: O47, C14, O52

1. Introduction

The club convergence hypothesis states that economies can be divided into convergence clubs characterised by a club-specific steady state in the long run. Several methods have been developed to identify convergence clubs. The most successful were Phillips and Sul (2007), who developed an algorithm (hereafter referred to as the PS approach) based on the relative convergence concept. Unlike previous methods for identifying convergence clubs, the PS approach considers heterogeneities in technical progress across countries and over time and is simple to implement. Furthermore, the PS approach has no assumptions regarding the trend stationarity or stochastic non-stationarity of the analysed time series and is robust to the presence of structural breaks. Consequently, the PS approach has been used frequently in recent years to look for convergence clubs, not only in the context of economic indicators but also in other scientific disciplines (Tomal, 2024).

A disadvantage of the PS approach is the lack of information regarding the time variability of the convergence process. Arestis et al. (2017) note that the speed of

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convergence can change dynamically over time due to structural and political changes in economies, especially during crises. Johnson (2020) also highlights that the forces driving convergence can fluctuate over time resulting in a shift in the speed of convergence. Finally, Corrado et al. (2005) found that convergence clubs also varied with time. The latter authors proposed an n-year rolling window method to detect the time-varying structure of the convergence clubs. Similarly, Montanés and Olmos (2013) using the PS approach investigated the effect of changing the end of the sample on the results of cluster analysis. In the context of evolving convergence clubs over time, Phillips and Sul (2009) highlighted that this situation may arise because individual economies may transition from one club to another due to their differences in performance, exposure to global technology, internal political shocks or social conflicts. However, methods that examine the temporal variance of convergence clubs based on subsamples have a fundamental drawback. They do not identify the number, size, and structure of convergence clubs at each point in time analysed, but only for a given sub-period. Therefore, this study aims to propose a new time-varying method for identifying convergence clubs that enables the identification of the unique composition of clubs at each point in the time series. The value of the proposed method is demonstrated through a case study.

2. The new method

The basis of the PS approach is the log t regression test, which, despite its many advantages, does not allow identification of the speed of the convergence/divergence process in each period studied. Johnson (2020) removed this limitation by conceptualising the local linear version of the log t regression test as follows:

$$\log\frac{H_1}{H_t} - 2\log(\log t) = m(\log t) + u_t$$
$$t = [rT], [rT]+1, \dots, T$$
$$H_t = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\log y_{it}}{\frac{1}{N}\sum_{i=1}^{N}\log y_{it}} - 1\right)^2 \tag{1}$$

where $\log y_{it}$ is a log variable of interest, H_t is the cross-sectional variance, r is a parameter intended to trim a certain number of initial observations in order to draw more attention to the rest of the sample. Phillips and Sul (2007) suggested using $r = 0.3$ for $T \leq 50$ and $r = 0.2$ for $T \geq 100$.

$m(\log t)$ is a smooth function whose first derivative indicates the speed of convergence in time t, in particular $2 > m'(\log t) \geq 0$ stands for convergence in growth rates while $m'(\log t) \geq 2$ for convergence in levels. Loader (1999) pointed out that for a fitting point x an estimator of $m'(x)$ is equal to $\frac{\sum_{i=1}^{n} w_i(x)(x_i - \bar{x}_w)y_i}{\sum_{i=1}^{n} w_i(x)(x_i - \bar{x}_w)^2}$ where y_i is the dependent variable, $\bar{x}_w = \sum_{i=1}^{n} w_i(x)x_i / \sum_{i=1}^{n} w_i(x)$ and $w_i(x) = k(x_i - x)/h(x)$. $k(x)$ is a kernel function, which ensures that observations closer to point x have a higher weight. $h(x)$ is the bandwidth, which determines the number of observations taken into account during estimation for point x. Johnson (2020) also indicated that a suitable method for constructing confidence intervals for $m'(\log t)$ is a non-overlapping block bootstrap due to the high persistence in the residuals from Eq. (1).

Given the above, the standard PS approach algorithm for identifying convergence clubs can be modified accordingly using the local linear log t regression test to obtain club convergence analysis estimates at each time t. The new algorithm at each time t includes the following steps:

- Step 1. *Sorting*. Sorting the units in the panel in descending order according to the observations at time t.
- Step 2. *Core group formation*. Starting from the units with the highest values of the variable from step 1, look for the first pair of units for which the lower limit of the 95% confidence interval for $m'(\log t)$ is greater than or equal to zero, i.e., $m'(\log t)^* \geq 0$. In order to estimate $m'(x)$, the shape of the kernel function $k(x)$ and the bandwidth $h(x)$ must be defined. In this algorithm, a tricube kernel was used and the bandwidth was assumed to be a window containing 40% of the observations. To generate the confidence intervals, the circular block bootstrap (1000 replications) was applied, which accounts for the so-called boundary effect. The block length was chosen as $[N^{\frac{1}{3}}]$. Subsequently, another unit is added to the identified pair of converging units and the convergence condition is checked. If collectively for these units $m'(\log t)^* \geq 0$ the unit is added to the core group and another unit is checked until $m'(\log t)^* < 0$.
- Step 3. *Club membership*. One by one, further units from the complementary core group are added to the core group from step 2 and if any of them $m'(\log t)^* \geq 0$ an updated core group is created. Each time, the convergence condition for the updated core group is checked. If adding a new unit would result in the condition not being met, that unit is removed from the updated core group. After analysing all units from the complementary core group, the updated core group serves as the first convergence club.
- Step 4. *Recursion and stopping*. For the remaining units not included in the club from step 3, steps 2 and 3 are repeated to further divide these units into convergence clubs. Units that do not belong to any convergence club form a group of units that diverge towards their own steady states.

3. A case study

To demonstrate the value of the new algorithm, a case study was performed by analysing club convergence in the gross domestic product (GDP) per capita based on purchasing power parity for 26 European countries from 1995 to 2021. Data are in constant 2017 international dollars and were obtained from the World Bank. Prior to the convergence analysis, a trend component was extracted from each time series using a boosted Hodrick-Prescott filter (Phillips & Shi, 2021).

Table 1 and Figure 1 present the results of the club convergence analysis. The standard PS approach identified two clubs, the first consisting of four countries, and the second consisting of 22. Table 1 also presents the results using the proposed time-varying framework at the beginning, middle, and end of the study period. The number of clubs, their sizes, and their structures vary over time. Interestingly, Club 1 in 2021 is identical to Club 1 from the PS approach. This situation may be due to the fact that the standard PS algorithm sorts the units in the panel according to the last observation in the first step. The new algorithm sorts the units according to the point in the time series currently being examined; therefore, when the last time period is studied, the sorting of the data is the same as for the PS approach. The problem of data sorting in the PS approach was highlighted by Haupt et al. (2018), who found that the sorting step affected core group formation. Ultimately, there is convergence in the growth rates in all identified clubs using both the PS and time-varying algorithms.

Table 1. Club convergence analysis results

The PS approach	$\hat{b}$	$t_{\hat{b}}$
Club 1: Luxembourg (LUX), Ireland (IRL), Lithuania (LTU), Poland (POL)	0.05	1.72
Club 2: Denmark (DNK), Netherlands (NLD), Austria (AUT), Sweden (SWE), Germany (DEU), Belgium (BEL), Finland (FIN), France (FRA), Cyprus (CYP), Italy (ITA), Czechia (CZE), Slovenia (SVN), Estonia (EST), Spain (ESP), Portugal (PRT), Hungary (HUN), Latvia (LVA), Slovakia (SVK), Romania (ROU), Croatia (HRV), Greece (GRC), Bulgaria (BGR)	0.15	32.35
The new method (year 2003)[‡]	$m'(\log t)$	$m'(\log t)^*$
Club 1: IRL, DNK, NLD, AUT, BEL, ITA, DEU, SWE, FIN, FRA, ESP, CYP, SVN, CZE, EST, SVK, LTU, LVA, ROU	0.72	0.57
Club 2: GRC, PRT, HUN, HRV, POL, BGR	0.39	0.37
Divergent unit: LUX	–	
The new method (year 2012)	–	
Club 1: LUX, IRL, AUT, DEU, LTU	0.41	0.13
Club 2: NLD, DNK, SWE, BEL, SVK, POL	0.34	0.27
Club 3: FIN, FRA, CZE, EST, HUN, LVA, ROU	0.74	0.36
Club 4: ITA, ESP, SVN, BGR	0.56	0.46
Club 5: CYP, PRT, HRV	1.58	0.85
Divergent unit: GRC	–	
The new method (year 2021)	–	
Club 1: LUX, IRL, LTU, POL	0.06	0.03
Club 2: NLD, AUT, SWE, DEU, BEL, FIN, CYP, CZE, SVN, EST, HUN, LVA, SVK, ROU, HRV, BGR	1.35	1.16
Club 3: ITA, ESP, PRT, GRC	0.53	0.02
Divergent group: DNK, FRA	–	

Note: $\hat{b}$ is the estimated convergence speed parameter, $t_{\hat{b}}$ is the value of a one-sided t-test for $\hat{b}$. The null hypothesis of convergence is rejected at 5% if $t_{\hat{b}} < -1.65$. $2 > \hat{b} \geq 0$ implies convergence in growth rates. [‡]The first eight observations are truncated.
Source: own study.

Figure 1. Club convergence analysis results on the map

Note: Each colour represents a separate convergence club and, in the case of one country, a divergent unit.
Source: own elaboration.

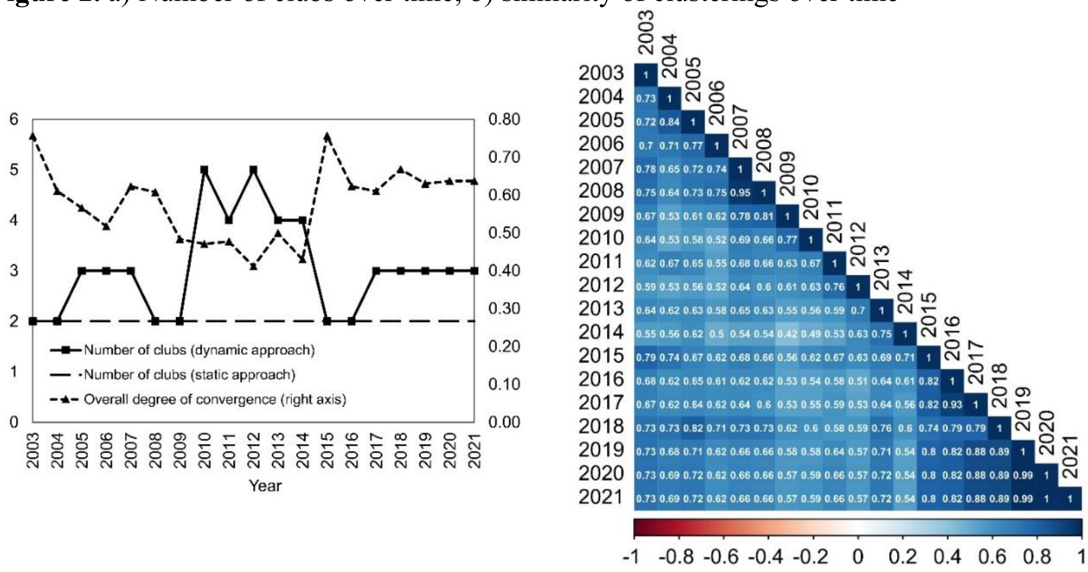
Figure 2a shows that the number of convergence clubs between 2003 and 2021 ranged from two to five, with the lowest rate of convergence across the panel occurring between 2010 and 2012. This period had the highest number of convergence clubs, but on the other hand, the speed of convergence within clubs was higher than at the beginning and end of the time series (see $m'(\log t)$ in Table 1). This situation may have been a consequence of the financial crisis of 2007-08 and the European debt crisis of 2009-10. These results are in line with Mazzola and Pizzuto (2020), according to which the 2007-08 crisis increased the divergence between groups of similar countries in Europe and, at the same time, accelerated the convergence within these groups.

Figure 2a also shows the overall degree of convergence in the sample in each year studied, calculated according to the formula (Hobijn & Franses, 2001):

$$d = \sqrt{\frac{\sum_{i=1}^{N}\sum_{j\neq i}^{N}\delta_{i,j}}{n(n-1)}} \quad (2)$$

where $\delta_{i,j}$ takes the value 1 if countries i and j are in the same club and 0 otherwise. In this approach, the measure d adopts a value between 0 and 1, with a higher value indicating a higher degree of convergence. As can be seen in Figure 2a, the value of the above measure is closely related to the number of clubs, and its smallest magnitudes can be observed between 2010 and 2014.

Figure 2. a) Number of clubs over time; b) similarity of clusterings over time



(a) (b)

Source: own elaboration.

Finally, the degree of similarity of clusterings obtained using the proposed time-varying algorithm is checked using the cluster correlation measure (Hobijn & Franses, 2000):

$$rc_{a,b} = \sqrt{\frac{\sum_{i=1}^{N}\sum_{j\neq i}^{N}\delta_{i,j}^{a}\delta_{i,j}^{b}}{\sqrt{\sum_{i=1}^{N}\sum_{j\neq i}^{N}\delta_{i,j}^{a}}\sqrt{\sum_{i=1}^{N}\sum_{j\neq i}^{N}\delta_{i,j}^{b}}}} \quad (3)$$

where $\delta_{i,j}^{a}$ ($\delta_{i,j}^{b}$) takes the value 1 if countries i and j are in the same club in the clustering a (b) and 0 otherwise. If clusterings a i b are identical then $rc_{a,b}$ is equal to 1. When $rc_{a,b} = 0$ then not a single pair of countries is in the same convergence club in both clusterings. The results revealed that the lowest similarity of clustering occurred between 2009 and 2014, and the highest between 2020 and 2021 (see Figure 2b). In general, since 2017, there has been noticeable stability in both the number of clubs and their internal structures.

Finally, in order to demonstrate the value of the proposed algorithm, an additional case study was performed for the data used by Phillips and Sul (2009) on income in 152 countries between 1970 and 2003 (see Figure A1 in Appendix A).

4. Conclusion

This study proposes a new method to identify convergence clubs over time by modifying the standard PS algorithm using the local linear version of the log *t* regression test. An analysis of convergence in GDP per capita among selected European countries revealed that the number, size, and structure of convergence clubs vary over time.

The method proposed in this article could be the subject of further research. In particular, future work may focus on the choice of bandwidth, which in this work was set at 40% of observations. Further, Morgan et al. (2024) suggested that in a dynamic algorithm that identifies convergence clubs, an invariant core group should be set in each successive iteration.

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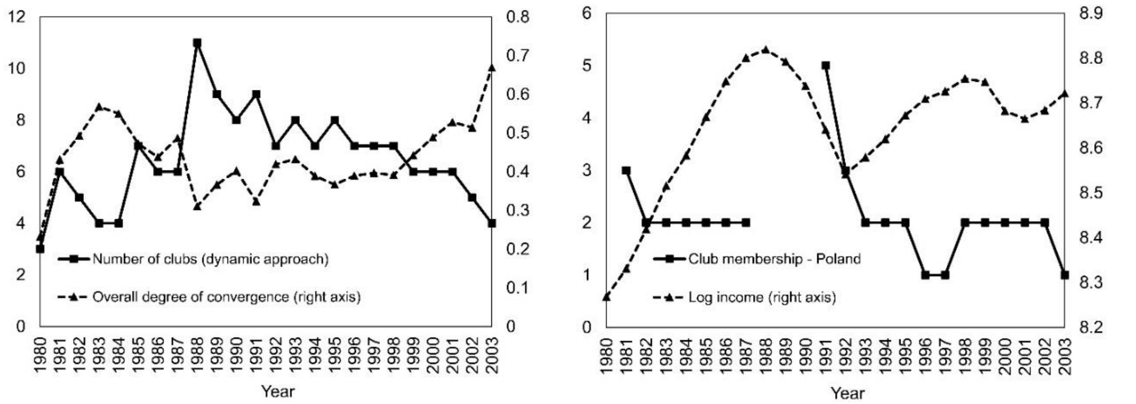
Appendix A

A dynamic analysis of club convergence using the proposed method for the data used in Phillips and Sul (2009) shows that the number of clubs oscillated between 3 and 11 during the period studied (see Figure A1a). The highest number of clubs appeared in the late 1980s and early 1990s, which may be due to the collapse of the Soviet Union and the change of economic system in many European countries. Phillips and Sul (2009) analysing the transitions of economies between clubs noted that there is a strong U-shaped income pattern for Central and Eastern European countries. Taking Poland as an example, it can be seen that changes in income strongly shaped the country's membership to a convergence club. In particular, the decline in income during the economic transition caused Poland to move to club 5, but this situation has changed in the following years, in which Poland returned to club 2 or 1 (see Figure A1b).

Figure A1. a) Number of clubs over time; b) Poland's transition between clubs based on Phillips and Sul (2009) data



(a) b)

Note: Interruptions in club membership mean that Poland did not belong to any club during these years.
Source: own elaboration.