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Shadow subscription for online video rental stores and affine systems

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Abstract
This article proposes a framework for evaluating whether a pricing mechanism that lowers marginal prices as consumption increases can outperform traditional pricing schemes—such as per item fees, subscriptions, or combinations of both—for firms offering zero marginal cost goods that cannot be easily resold (e.g., films). The problem is addressed through a computational agent-based model. Substantial improvements in revenue and consumer surplus are observed between the optimal price-by-quantity curves and the conventional alternatives (transaction fees, subscription and a combination of both) for the constructed populations of consumers. Further research avenues are suggested.

Keywords: monopoly pricing, platforms
JEL Classification Codes: L5, L12, L82, L86

1. Introduction

This article proposes a framework for evaluating whether a pricing mechanism that lowers marginal prices as consumption increases can outperform the traditional pricing schemes—such as per item fees, subscriptions, or combinations of both—for firms offering zero marginal cost goods that cannot be easily resold (e.g., films). This category includes Internet based platforms like Netflix, HBO, Filmin, YouTube, and Substack. The term "shadow subscription" refers to the specific case in which the price–quantity function has an asymptotic maximum that caps total payment, providing users with psychological reassurance about the highest amount they might pay.

The literature on optimal pricing in the Internet economy is extensive and mathematically

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complex: the existence of several income sources (e.g. consumer charges and advertisement) leads to bilateral pricing problems (Sanchez-Cartas and Leon, 2021). Most studies focus on membership fees (subscriptions), but transaction fees (fixed cost by item delivered) are important because they make multisided markets more contestable (see Hagiu and Wright, 2019; Caillaud and Jullien, 2003).

For informational goods, sold to identified users, prices can be tailored to the quantity demanded. The literature on monopoly pricing (Tirole, 1988) typically supposes a single price, or prices depending on observable characteristics. Deriving an optimal price-by-quantity curve for monopolists facing diverse demands by consumers with hidden heterogeneity is rarely studied, except in stylized examples: Wilson (1988) examines two consumer groups. Quantity discounts are discussed in the "economic order quantity (EOQ) problem" (Pereira and Costa, 2015), but this literature is focused on the buyer's perspective rather than that of the monopolist. A recent paper by Anderson and Bedre Defolie (2024) presents a tractable model of hybrid platforms that simultaneously host third-party sellers and sell their own products and show that they can strategically reduce variety and increase prices, adversely affecting welfare.

Given the importance of individual heterogeneity to address this problem, a computational agent-based model (Sanchez-Cartas, 2018) is used. Finally, the conclusions suggest some avenues for further research.

Replication materials for this article can be found at https://osf.io/tyega.

2. Modelling framework

Consider a company that offers a catalogue of N exclusive goods, hereafter referred to as “films,” to a market of M consumers. Rather than modelling consumer preferences through a utility function defined over portfolios of purchased films, the standard auction theory approach is adopted: each consumer attributes reservation values to every film. For tractability, we assume that the monopolist is either alone in the market, or she can ignore the strategic behavior of other sellers and simply suppose an exogeneous distribution of reservation prices over the films.

Both film-level and consumer-level price discrimination are ruled out: the platform cannot charge different prices across consumers, nor can it set distinct prices for different films. However, it can still employ a tailored price-quantity function. Formally, this is represented by $\mathrm{p(q)}$, which maps the number of films demanded to their corresponding aggregate cost. Incentive compatibility implies that $\mathrm{p(q)}$ is monotonic and sub additive (there is no incentive to create “sock puppet” accounts to buy goods separately):

$$\mathrm{p}(\mathrm{q}_1 + \mathrm{q}_2) \leq \mathrm{p}(\mathrm{q}_1) + \mathrm{p}(\mathrm{q}_2) \tag{1}$$

The i film is supposed to have a value $\mathrm{v}_{\mathrm{m,i}} \geq 0$ for the consumer m (negative valuations are excluded because the consumer can simply choose not to watch the film). In practice, given that all films are alike (except for the valuations that each consumer assigns to them) it is convenient to use $\sigma_m(.)$ the permutation that orders each row by decreasing valuation, so:

$v_{m,\sigma_m(s)} \geq v_{m,\sigma_m(s+1)}$. The cumulative consumer value of buying the most valuable films from 1 to q is:

$$V_m(q) = \sum_{s=1}^{q} v_{m,\sigma_m(s)} \tag{2}$$

Each consumer orders the number of films q that maximizes her consumer surplus (the consumer valuation of the goods minus the total cost). Given v and p(.), the individual demand of consumer m is:

$$q(m) = \text{argmax}_q\{V_m(q) - p(q)\} \tag{3}$$

where $V_m(0) = p(0) = q(0) = 0$. The online video rental store revenue is:

$$Revenue = \sum_{m=1}^{M} p\big(q(m)\big) \tag{4}$$

The total value (or welfare) created by the online store is the sum of the total consumer surplus plus the revenue. Potential value is the maximum socially attainable value (e.g. obtained when each consumer obtains the films she wants for free). Potential value for agent m is simply $V_m(N)$, and for the entire market is $\sum_{m=1}^{M} V_m(N)$. For comparability all value measures in this paper (revenue, consumer surplus and the sum of both, named "total value") are normalized by the potential value.

This general framework makes it possible to examine alternative pricing systems, since such systems can be conveniently expressed as a price–quantity function:

1. Subscription (*Subs*) allows the consumer to watch all the films she values over zero at a fixed price:

$$p_{Subs}(q) = c_{Subs} \tag{5}$$

2. Pay-per-item (*Item*) with a fixed cost (p) by film implies q films are bought by pq:

$$p_{Item}(q) = pq \tag{6}$$

3. Combined subscription and pay-per-item (*Subs and Item*) where the consumer chooses the cheapest way to satisfy her demand:

$$p_{Subs\ and\ Item}(q) = \min\{pq, c_{Susc}\} \tag{7}$$

4. The incentive compatible (monotonic and sub additive) price-quantity curve computed by optimization (*Curv*) is named $p_{Curv}(q)$.

3. The subscription system

Subscription appears to be the dominant pricing system for platforms that sell zero marginal cost goods. Under standalone subscription the monopolist does not need any information on the distribution of value across films because the complete catalogue is bought: the sum of the reservation values of all the positive valued films by each consumer is the only relevant information needed.

Suppose q(V) is the distribution of the total value of the catalogue across consumers and Q(.) is the cumulative probability distribution. Then $Q(\infty)=\int_0^\infty q(V)dV = 1$ and the total value of the catalogue is $\bar{V} = \int_0^\infty Vq(V)dV$. The monopolist problem under subscription is:

$$max_{p>0}\, p \int_p^\infty q(V)dV = max_{p>0}\, p[1 - Q(p)] \tag{8}$$

The existence of a maximum is guaranteed for any distribution with finite mean, because for p>1:

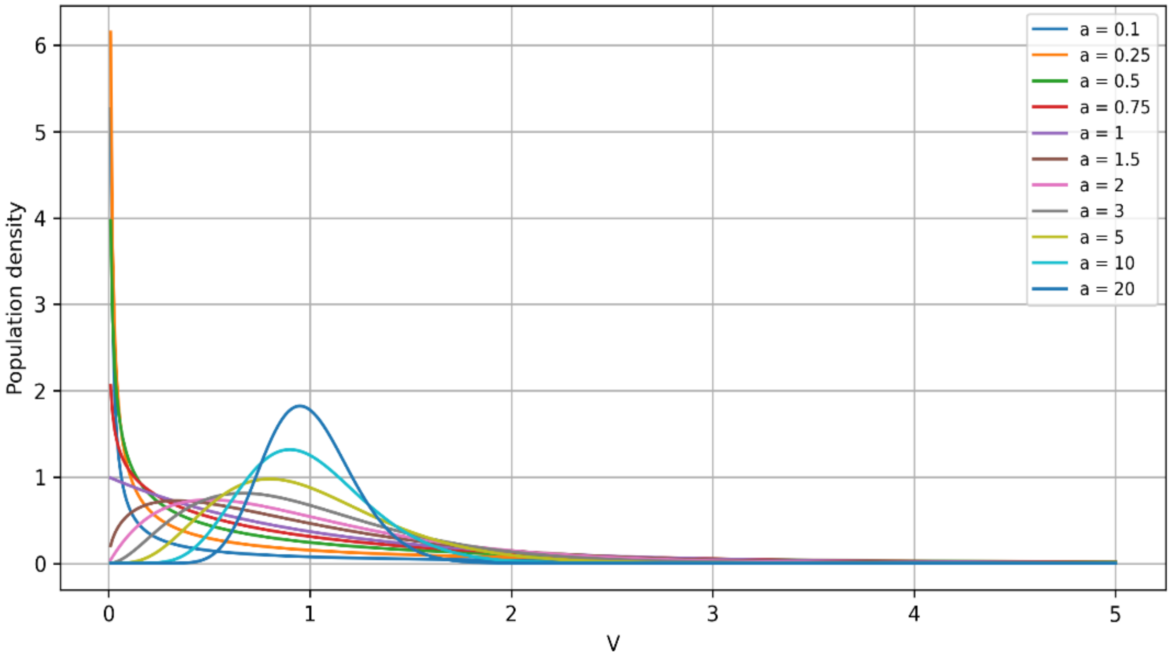
$$p \int_p^\infty q(V)dV = \int_p^\infty pq(V)dV \leq \int_p^\infty Vq(V)dV = \bar{V} \tag{9}$$

Being positive and finite, the tail $\int_p^\infty Vq(V)dV$ tends to zero as $p \to \infty$. That means that the maximized expression $p\int_p^\infty q(V)dV$ is 0 in p=0 and $p \to \infty$, and it has an interior maximum.

If a parametric family is chosen for $q(V)$, Eq. 8 enables the explicit computation of the monopolist's optimal subscription price and the corresponding revenue outcomes. The gamma distribution is adopted due to its analytical simplicity and its connection to income distributions (Salem and Mount, 1974; McDonald, 2008), that are the natural starting point for modelling willingness to pay. Without loss of generality, the mean of the distribution is normalized to 1—consistent with expressing monopolist revenue as a percentage of potential value ($\bar{V} = 1$)—by setting the scale parameter equal to the inverse of the shape parameter. The shape values considered are $a \in \{0.1, 0.25, 0.5, 0.75, 1, 1.5, 2, 3, 5, 10, 20\}$, with the resulting value distributions shown in Figure 1.

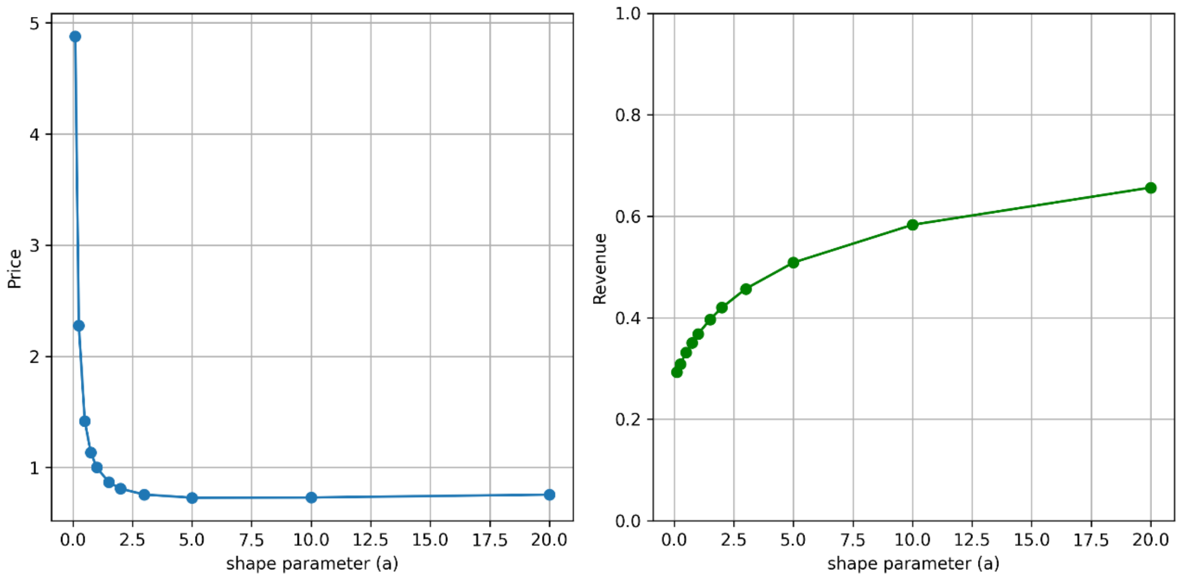
Figure 2 shows how the monopoly subscription price (left panel) and the normalized revenue (right panel) vary with the scale parameter a. As the distribution becomes more concentrated, revenue rises; in the limit (when all consumers share the same valuation), the monopolist can extract the entire surplus by setting the price infinitesimally below the common reservation value. Consequently, any profitable deviation from subscription pricing relies on the heterogeneity in consumers' valuations of the catalogue.

Figure 1. Gamma density distributions



Notes: For selected shape parameters (a) and unit mean (the scale parameter is 1/a).

Figure 2. Standalone subscription: optimal monopolist price and revenue



Notes: Revenue maximizing price (left panel) and revenue as a fraction of potential value (right panel) for the subscription system, for selected values of shape parameter (a), and fixed mean of the distribution equal to one (scale parameter is 1/a).

4. Numerical analysis

This paper investigates whether an optimized price–quantity function can yield more revenue for a monopolist supplying goods with zero marginal cost than the traditional pricing schemes. To explore this question, simulated samples of artificial consumers are constructed using the results from Section 3: the consumer valuations of the entire catalogue are generated from the gamma distribution with shape parameter a a=0.1 and mean 1. Under this specification, the subscription mechanism captures approximately 30% of the total potential value, thereby leaving scope for alternative pricing strategies to extract more revenue.

Once the total value of the catalogue ($\mathrm{V(m)}$) has been randomly generated for consumer m, the distribution of that value across the N films is still a free parameter that can be used to create an artificial population where revenue extracted by $\mathrm{p_{Curv}(.)}$ is substantially higher than revenue extracted by $\mathrm{p_{Subs\ and\ Item}}$ $(.)$. To this end $\mathrm{V(m)}$ is distributed across films by this algorithm:

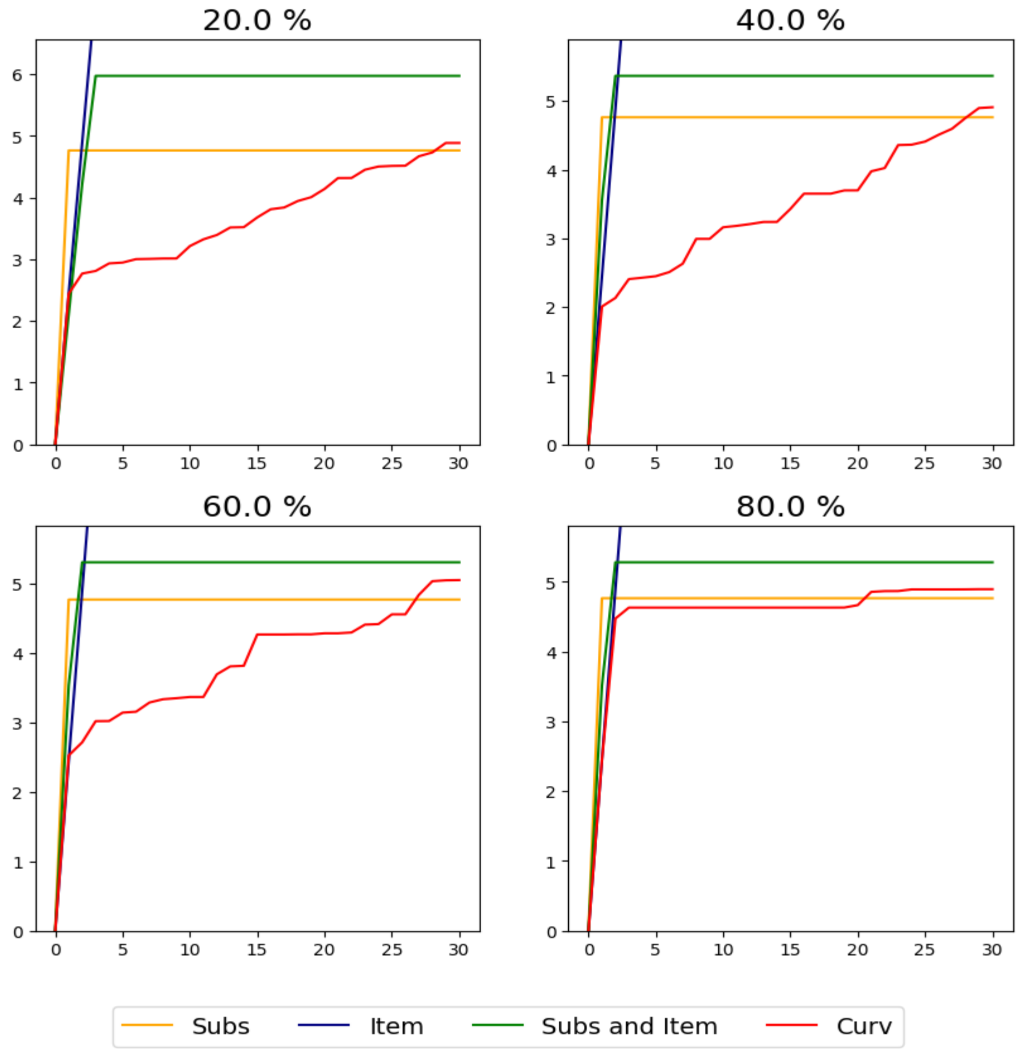
1. For consumers whose total valuation of the entire catalogue surpasses the individual subscription price established analytically in Eq.8 ($\mathrm{V(m)} \geq \mathrm{c}_{Subs}$), this aggregate value is allocated uniformly across all films. Such consumers are named "whales".
2. Consumers whose valuation of the entire catalogue is below the optimal subscription ($\mathrm{V(m)} < \mathrm{c}_{Subs}$) are randomly assigned into two groups:
   a. "Single film" consumers (with probability p_{single}): all the value $\mathrm{V(m)}$ is put in the most valued film.
   b. "Intermediate" consumers (with probability $1 - p_{single}$): a random number n is drawn uniformly from 1 to N, and the consumer's total value (as drawn from the gamma distribution) is evenly distributed across the first n (most valued) films.

The *Subs and Item* mechanism is expected to optimize the subscription fee mostly by maximizing revenue extraction from the "whales", that having a diffused interest across the films in the catalogue will chose the subscription over buying N films individually. On the other hand, the per-item leg of *Subs and Item* is expected to extract as much value as possible from the "single-film" consumers. This leaves the "intermediate" consumers largely unaddressed. Consequently, the more flexible optimal incentive-compatible pricing curve is expected to raise higher revenue from this population. Different sizes of the "single film" consumer population are considered: $p_{single} \in \{0.2,0.4,0.6,0.8\}$.

The optimized curves (in this discretized context "curve" means a vector of dimension N-1) for the four considered pricing mechanisms are presented in Figure 3 for N=30, M=2000. The other cases ($N \in \{20,30,40\}$, $\mathrm{M} \in \{150000, 200000\}$) are included in the replication materials as a sensitivity analysis.[1]

[1] The starting point to understand the script and its non-graphic outputs is the ReadMe.docx file in the replication materials.

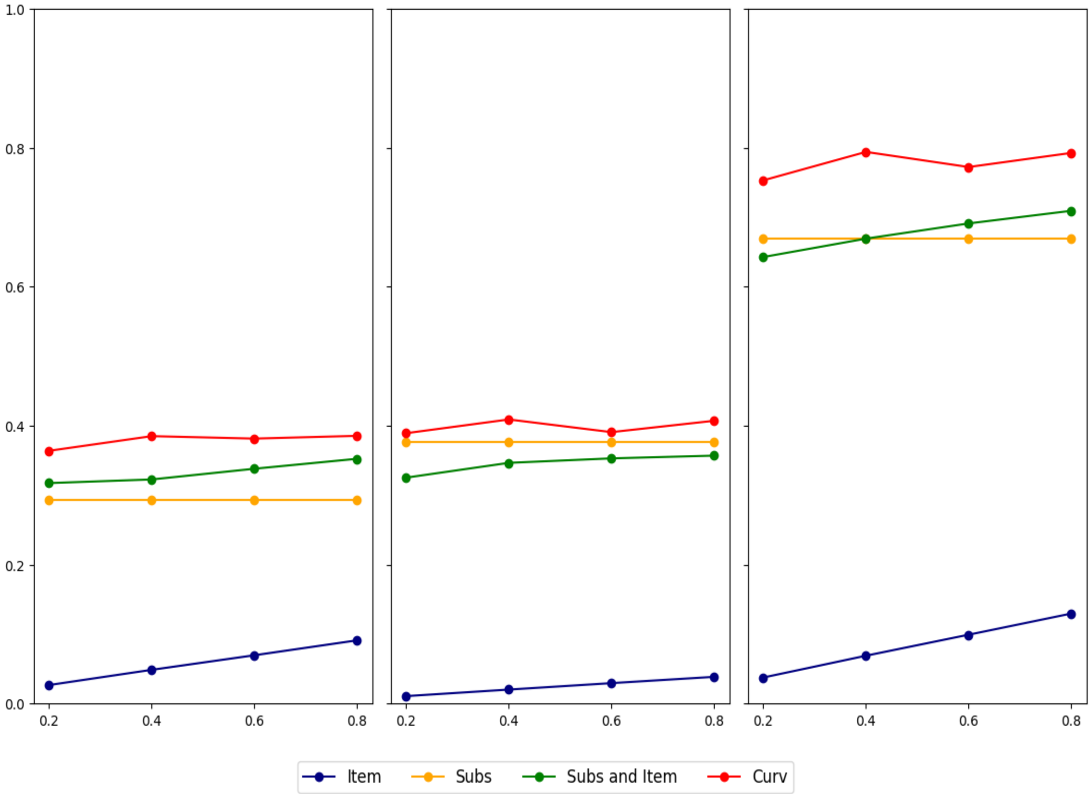
Figure 3. Total price by quantity curves for different pricing systems (N=30 films, N=200,000 consumers)



Notes: Optimal price-to-quantity curves for the considered mechanisms. Each panel (see panel titles) represents a different value of p_{single}.

The free parameters for each mechanism are chosen by optimization, being Curv the case where the complete curve is a free parameter (only limited by the monotonic and subadditivity constraints). The COBYLA algorithm from Python's Scypi has been used for optimizing the curve under monotonicity and subadditivity. Unfortunately, the simple introduction of the subadditivity constraints was not enough to produce truly sub-additive curves (a defect of COBYLA) and the constraint had to be manually modified (strengthened) by a parameter $\lambda \geq 1$, so that $\lambda * p(q_1 + q_2) \geq p(q_1) + p(q_2)$, until the curves became sub-additive (as checked independently in the script). Some violations of monotonicity that have been detected are insignificant (beyond the fifth decimal place).

Figure 4. Revenue, consumer surplus and total value by pricing mechanism (N=30, M=200.000)



Notes: Each panel represents a part of the value waterfall: the left panel is revenue, the middle panel is consumer surplus, and the right panel is total value. Each line represents a pricing mechanism. Values in the x-axis represent the different values of p_{single}.

The three panels of Figure 4 show the value waterfall under each pricing system. The left panel presents the monopolist's revenue, the middle panel the consumer surplus, and the right panel presents the sum of both, that is, total value (or welfare). All results are presented as a fraction of potential value in a common zero-to-one scale for the y-axis.

Regarding revenue, *Subs* extracts around 30% of potential value (see the analytical derivation in Section 3). The revenue raised by *Item* grows almost linearly with p_{single} since it primarily extracts value from "single-film" concentrated consumers, and its revenue increases with the abundance of this segment. *Subs and Item* extracts revenue in line with the sum of its two components. Finally, *Curv* raises substantially more revenue than the classical mechanisms.

Because each mechanism is optimized for revenue, the resulting structure is clear: the combined *Subs and Item* mechanism yields higher revenue than either component alone, and *Curv* outperforms the classical mechanisms (optimizing over a larger support set).

The second panel indicates that consumer surplus under *Subs* remains constant, whereas for *Item* it increases almost linearly with p_{single}. The *Subs and Item* mechanism generates lower consumer surplus than *Subs* does—a pattern that would be impossible for revenue but it is feasible for the consumer surplus, since the monopolist does not optimize it.

As expected by construction, *Curv* produces the highest revenue, and it does so by a substantial margin. Notably, it also yields the *highest consumer surplus*. This suggests that by targeting all consumer segments with greater precision, *Curv* stimulates additional transactions that benefit both the monopolist and consumers. That said, the consumer-surplus advantage of *Curv* is not an analytical result; further theoretical work is required to characterize the conditions under which it holds. Nonetheless, this numerical example already demonstrates that the revenue gains induced by *Curv* need not come at the expense of consumer welfare.

Finally, all of this translates into the results for total value, where *Curv* is the pricing system delivering the highest total welfare. The sensitivity analysis Appendix in the replication materials shows the robustness of the results here presented.

5. Conclusions

This article proposes a framework for evaluating whether a pricing mechanism that lowers marginal prices as consumption increases can outperform traditional pricing schemes—such as per item fees, subscriptions, or combinations of both—for firms offering zero marginal cost goods that cannot be easily resold (e.g., films). This category includes Internet based platforms like Netflix, HBO, Filmin, YouTube, and Substack. The term "shadow subscription" refers to the specific case in which the price–quantity function has an asymptotic maximum that caps total payment, providing users with psychological reassurance about the highest amount they might pay.

For the constructed population substantial differences in revenue and total value (revenue plus consumer surplus) are found between the optimized curve and the classic pricing systems (transaction fees, subscription and a combination of both), and the superiority of the optimized curve is even more intense for total value (the comprehensive measure of welfare).

These findings raise an important puzzle: why are price-by-quantity schemes—such as those proposed in this article—virtually absent in the streaming market? One possibility is that, under many reasonable consumer valuation patterns, existing subscription and per-item pricing already capture revenues sufficiently close to the theoretical maximum. Another is that price-by-quantity mechanisms may impose excessive cognitive load on real-world consumers, reducing their practical appeal.

Nevertheless, this paper aims to encourage further research on the shadow subscription mechanism. Promising avenues include analyzing their performance under alternative valuation distributions, extending the framework to bilateral markets (incorporating advertising), and studying their implications in settings with strategic interaction among competing platforms. Additionally, the shadow subscription may prove especially valuable for market challengers: in contexts where consumers expect sporadic usage, lowering the artificial entry barriers created by flat rate subscriptions could facilitate adoption and accelerate user acquisition.

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