Economics and Business Letters
2(2),54-65, 2013

Spatiotemporal inflation dynamics in response to a monetary policy shock

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Received: 29 March 2013
Revised: 16 June 2013
Accepted: 17 June 2013

Abstract

This note characterizes the adjustment of the inflation rate towards a new steady-state, in response to a monetary policy shock. The sluggish nature of the adjustment is the straightforward outcome of the type of information dissemination one considers. Namely, firms will be grouped in clusters, and it will be assumed that information spreads faster within than between clusters.

Keywords: inflation dynamics, monetary policy, information diffusion, clusters of firms

JEL Classification Codes: E52, E31, D80

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1. Introduction

Macroeconomists have often emphasized that the economy tends to react sluggishly to policy shocks.[1] One probable cause for the existence of inertia in the reaction to policy measures is the stickiness on the diffusion of information; because firms do not share the same degree of attentiveness to the arrival of new information, the response to shocks acquires a gradual nature.[2]

This note approaches the sluggish response of the economic activity to a monetary policy shock that changes the long-run value of the inflation rate. The setting is one of monopolistic competition, where firms are organized in clusters within which information circulates fast. Between clusters, the contact among firms is more sporadic and the information does not spread so rapidly. The 'information space' will be conceptualized under the form of a sphere that is cut into a given

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Citation: Gomes, O. (2013) Spatiotemporal inflation dynamics in response to a monetary policy shock, *Economics and Business Letters*, 2(2), 54-65.

[1] See Sims (1998) for a thorough discussion on the sources and consequences of macroeconomic stickiness.
[2] This is the assumption that underlies the popular sticky-information macro model of Mankiw and Reis (2002).

number of locations or clusters, each one containing an identical number of firms.[3]

The current study has some points of contact with Gomes and Mendes (2011), in the sense that it relies on information diffusion in order to explain the impact of a monetary policy shock, but it departs on one important point: the concept of diffusion that is adopted in this note is not a straightforward notion of unconstrained dissemination of information. As stated, the information space is organized in clusters, and this puts us close to the ideas/technology diffusion mechanism explored in Lucas (2009) and Comin *et al.* (2012). This mechanism emphasizes that diffusion between locations occurs at a slower pace than diffusion inside each location.

The analysis gives attention both to the temporal and to the spatial dimensions of the propagation of shocks. The idea that time and space should be integrated in a same structure of analysis is not new either in Economics,[4] or in other scientific fields.[5]

The note is organized as follows. In section 2, one discusses the notion of information clusters and how this concept might be relevant to add to our knowledge about the response of the economy to policy shocks. Section 3 characterizes the behavior of firms in a monopolistically competitive market environment and describes how the potential to collect information is distributed across the market. Section 4 addresses the agents' expectations and derives an expression for the dynamics of the inflation rate. Section 5 presents a few generic results and explores a numerical example. Section 6 concludes.

2. The notion of information cluster

What does the proposed notion of information cluster effectively mean? Does this concept help in devising a more realistic and complete tool to understand the propagation of a policy shock across a population of firms? The argument underlying the analysis in this paper is that industries are composed by firms with different characteristics that allow them to be classified into a series of groups or clusters. Firms within each group or cluster have similar features and eventually have a more direct contact between them, what allows information to disseminate faster and with a higher degree of accuracy between firms within each group than between firms across groups.

By noticing that firms are not an undifferentiated mass of organizations for which information hypothetically diffuses in a homogeneous way regardless from the type of the firm, we take a step towards a more realistic process of information dissemination, allowing for a more comprehensive approach to the impact of policy shocks. A standard model of information diffusion is capable of explaining the inertia associated with the adjustment of firms to a new economic event; nevertheless, this characterization can be improved by introducing a structure for the way in which information disseminates. The contribution of this paper is precisely to offer such structure and to understand, through it, how the shape of observed fluctuations is decisively dependent on what makes information to diffuse one way or the other.

Under our interpretation, the elements that effectively determine the trajectories of macro variables in the short-run, as the result of a given disturbance that deviates the economy from its

[3] The specific shape assumed for the information space is useful because it places all firms in the exact same conditions relatively to the whole of the market.

[4] See, e.g., Desmet and Rossi-Hansberg (2009).

[5] See, e.g., Mistro *et al.* (2012) and Rodrigues *et al.* (2012), for spatiotemporal analysis in ecological systems.

steady-state, are those that are related to the structure of information dissemination: how firms are grouped, how fast the information spreads and how distant are the clusters of firms in terms of their capacity to share information. Although this note does not explore in detail all the potential of the proposed setting, the idea is that it can contribute to set a new framework to explain the diffusion of information in a macro environment that does not put all firms in a same level of analysis. Firms are different, share different features across them and taking a more detailed look on how they interact can only improve the perception we might gain on the way business fluctuations evolve.

To conceive some kind of classification that arranges firms into groups or clusters is apparently wiser than to treat them as being equal at various dimensions, including the easiness with which they share and absorb information. The criteria to form such clusters can be manifold. As an illustration, we could for instance take the classification of firms proposed by Romero and Santos (2007), who present a typology that allows to place firms into seven different categories. The criteria to form these groups has to do with the location of suppliers, the location of sales markets, the position within the chain of value, and the technological potential of firms. To these we can add some more distinctive features, as the size of the firm, the legal status (state-owned or private owned) or whether the firm is an independent firm or a subsidiary of a larger company.

All the above characteristics furnish the data one needs to form a map of firms, a map that gives indication on the similarities and on the differences between productive units such that they will find it easy or hard to transmit and receive information flows. It is in the described sense that we approach information clusters and also the notion of information space.

3. The market, the firms and the diffusion of information

Take a monopolistically competitive market environment, where a large mass of firms behave optimally. Each productive unit will maximize profits, at each time period, by choosing an optimal desired price. The solution of this problem is[6]

$$p_t^* = p_t + \alpha y_t \tag{1}$$

In equation (1), p_t and y_t correspond, respectively, to the price level and to the output gap; the two variables are measured in logs and, thus, the same is also true for the desired price p_t^*. Parameter $\alpha > 0$ is a measure of real rigidities; it indicates the extent in which departures from potential output deviate the desired optimal price from the observed price level.

Let m_t represent money supply (also in logs), in order to add to the analysis a straightforward relation between the monetary aggregate and nominal output,

$$m_t = p_t + y_t \tag{2}$$

Further important insights on the economic structure relate to the interaction firms establish with each other, which allows information to be disseminated. One assumes that firms are

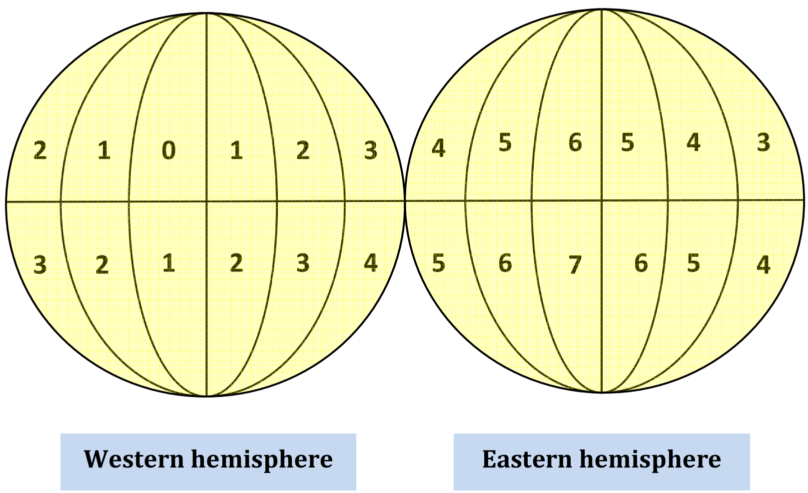
[6] See Blanchard and Kyotaki (1987) for details on the derivation of equation (1).

distributed in an informational space separated in a finite number of locations. This is a conceptual space that translates the easiness with which firms share relevant information.

To materialize the mentioned space, we conceive it as a sphere, over which firms are uniformly distributed. This informational globe can be split into multiple locations. A tractable way of dealing with the defined space is to cut it in half through an equator line and then to take as many meridian lines as the ones required to divide the space into a given number of identical sectors, i.e., of locations/clusters with a same area.

In a spherical space where locations are formed by assuming a north hemisphere, a south hemisphere, and a given number m of meridian lines, there will exist $4m$ different locations. Distance between locations will be set to 1 when a border line exists between them. As a result, it is straightforward to infer which is the distance between any two locations. Figure 1 elucidates how distances are measured, by considering an example with $m=6$. The numbers in the figure represent distances relatively to the location marked with a '0'.

Figure 1. Distance between locations (*m*=6)



Next, one stipulates a mechanism for information diffusion. Market relations induce a scenario where, in every period, each firm is in contact with a share $\theta \in (0,1)$ of the set of the firms in the market. These meetings occur randomly and they generate the transmission of information. When one firm, possessing relevant information, meets another firm that does not have yet knowledge on the new economic conditions, a transmission of information from the first to the second one occurs. Thus, value θ can be interpreted as the speed at which information disseminates.

Another parameter, $\delta > 0$, defines the extent in which the diffusion of information is more or less localized. The value of δ translates the importance of space by establishing that the probability of a firm at location ℓ meeting a firm at location $\tilde{\ell}$ is $\exp\left(-\delta\left|\ell-\tilde{\ell}\right|\right)$ times lower than the probability of meeting a firm in the same informational cluster.

4. Expectations and inflation dynamics

Assume a monetary policy shock, under the form of a change on the rate of growth of m_t.[7] This shock occurs at $t = \widetilde{t}$, but it will not be perceived instantly by all of the firms in the market. The agents who are able to understand the impact of the shock at some period $t \geq \widetilde{t}$, will be capable of formulating a rational expectation for the price level at date $t+1$. In the absence of uncertainty, perfect foresight implies $E_t^I p_{t+1}^* = p_{t+1}^*$. The agents who formulate this type of forecast take the designation of type *I* agents.

There is a second class of agents, those that have not accessed or processed the information on the shock at time *t*. These will form an evolutionary expectation such that the optimal price at *t*+1 should be equal to the price level at *t* plus the money growth rate that existed previously to the disturbance. The mentioned growth rate of money supply is specified as $\Delta m|_{\underline{\Omega}}$, with $\underline{\Omega}$ the information set available prior to the knowledge about the shock. Type *II* agents will form the expectation $E_t^{II} p_{t+1}^* = p_t + \Delta m|_{\underline{\Omega}}$.

The price level at *t*+1 will be a weighted average of the expectations formed at period *t* on the desired price, given the shares of agents of each one of the types mentioned above,

$$p_{t+1} = \frac{\sum_{\ell=1}^{4m} G_t(1,\ell)}{4m} E_t^I p_{t+1}^* + \frac{\sum_{\ell=1}^{4m} G_t(0,\ell)}{4m} E_t^{II} p_{t+1}^* \tag{3}$$

In expression (3), $G_t(0,\ell)$ and $G_t(1,\ell)$ stand for, respectively, the share of agents that have not yet accessed and that have already accessed the information on the monetary policy shock at period *t* and location ℓ.[8]

Taking into account the expectations' expressions and the optimal price in equation (1), one is able to rewrite equation (3) under the form of a Phillips curve, that reveals the existence of a positive contemporaneous relation between the output gap and the inflation rate,[9]

$$\pi_{t+1} = \alpha \frac{\sum_{\ell=1}^{4m} G_t(1,\ell)}{\sum_{\ell=1}^{4m} G_t(0,\ell)} y_{t+1} + \Delta m|_{\underline{\Omega}} \tag{4}$$

The suppression of the output gap in equation (4) through the use of relation (2) and the

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[7] We assume a permanent money supply shock. This means that the growth rate of money supply increases or decreases to a new value, not returning in a posterior time period to the initial level. The main implication of this assumption is that macroeconomic variables and, specially, the inflation rate that is here subject to a detailed analysis, will converge to a new steady-state (in the case of inflation, the steady-state value has correspondence on the rate of change of money supply). If the shock were temporary, one would observe a same type of adjustment process involving inertia, with one difference: after the effect of the shock faded out, the original steady-state would be reestablished.

[8] Obviously, $G_t(1,\ell) = 1 - G_t(0,\ell), \forall \ell = 1,...,4m.$

[9] Note that $\pi_t := p_t - p_{t-1}$ is the inflation rate at time *t*.

posterior application of first differences, leads to a difference equation representing the dynamics of the inflation rate,[10]

$$\pi_{t+1} = \frac{1}{\sum_{\ell=1}^{4m} G_t(0,\ell) + \alpha \sum_{\ell=1}^{4m} G_t(1,\ell)} \left[\frac{\sum_{\ell=1}^{4m} G_{t-1}(0,\ell) \sum_{\ell=1}^{4m} G_t(1,\ell)}{\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)} \pi_t \right. \\ \left. + \alpha \sum_{\ell=1}^{4m} G_t(1,\ell) \Delta m|_{\overline{\Omega}} - 4m \frac{\sum_{\ell=1}^{4m} G_t(1,\ell) - \sum_{\ell=1}^{4m} G_{t-1}(1,\ell)}{\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)} \Delta m|_{\overline{\Omega}} \right] \quad (5)$$

with $\overline{\Omega}$ the after-shock information set.

5. General properties of the inflation adjustment process and numerical illustration

As formulated, the process of interaction between firms implies the following probability of not accessing the information at period *t*+1 conditional on not having the information in period *t*, for any location $\tilde{\ell}$,

$$G_{t+1}(0,\tilde{\ell}) = G_t(0,\tilde{\ell}) \left[\frac{\sum_{\ell=1}^{4m} G_t(0,\ell) \exp\left(-\delta \left|\ell - \tilde{\ell}\right|\right)}{\sum_{\ell=1}^{4m} \exp\left(-\delta \left|\ell - \tilde{\ell}\right|\right)} \right]^{\theta} \quad (6)$$

Equation (6) describes the process of information diffusion. The corresponding properties are trivial, and mimic the ones from the analysis undertaken by Comin *et al.* (2012) relating the diffusion of technology: independently of the initial distribution (in our case, distribution of information), the fraction of firms gaining access to the resource rises faster in locations closer to the source and the effect of distance vanishes over time, i.e., equation (6) implies that $G_t(0,\tilde{\ell}), \forall \tilde{\ell} = 1,\dots,4m$, converges asymptotically to zero.

Steady-state properties can be characterized as follows.

Proposition 1 *In the steady-state, the inflation rate is* $\pi^* = \Delta m|_{\overline{\Omega}}$.

Proof As information disseminates, all firms in the market will end up by gaining awareness of the impact of the shock, i.e., $\lim_{t\to\infty} G_t(0,\ell) = 0, \forall \ell$. Thus, the steady-state can be defined as the long-run state in which $\pi^* := \pi_{t+1} = \pi_t$ and where the process of diffusion of information is completed. Applying these two conditions to equation (5), it is straightforward to arrive to the result in the proposition ■

[10] The steps required for the derivation of this equation are presented in appendix.

Note, as well, the following result,

Proposition 2 *The steady-state is asymptotically stable.*

Proof The differentiation of the r.h.s. of equation (5) with respect to π_t yields

$$\frac{\partial \pi_{t+1}}{\partial \pi_t} = \frac{1}{\sum_{\ell=1}^{4m} G_t(0,\ell) + \alpha \sum_{\ell=1}^{4m} G_t(1,\ell)} \frac{\sum_{\ell=1}^{4m} G_{t-1}(0,\ell) \sum_{\ell=1}^{4m} G_t(1,\ell)}{\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)}$$

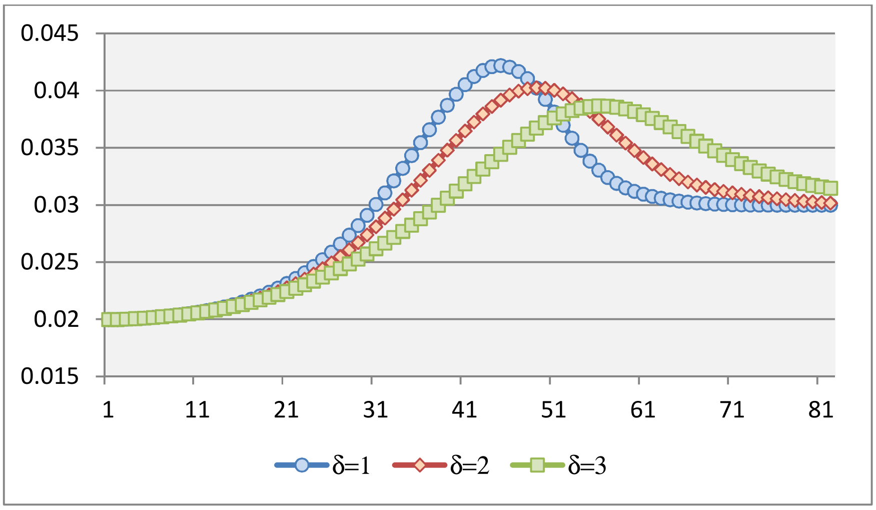
The evaluation of this expression in the steady-state gives the result $\left.\frac{\partial \pi_{t+1}}{\partial \pi_t}\right|_{\pi^*} = 0$. Since this derivative lies inside the unit circle, one confirms that the steady-state is asymptotically stable ■

The previous results indicate that once the monetary policy shock takes place, a new long-run inflation rate emerges and the system converges to it. The shape of the convergence process will depend on how information diffuses. This will be determined by the initial levels of information available at each one of the clusters and on the values of the various parameters.

To illustrate possible outcomes about the trajectories followed by the inflation rate after the shock, we consider a numerical example. The example takes $\Delta m|_{\Omega} = 0.02$, $\Delta m|_{\overline{\Omega}} = 0.03$, $m = 4$, $\theta = 0.1$, $G_{\tilde{t}}(0,\tilde{\ell}) = 0.8$ and $G_{\tilde{t}}(0,\ell) = 1, \forall \ell \neq \tilde{\ell}$. For the parameter that measures the importance of space, we assume three alternatives: $\delta = 1$, $\delta = 2$ and $\delta = 3$. Recall that the higher the value of δ, the more localized the diffusion of information is. A low value of δ represents a relatively homogeneous market environment; following the discussion in section 2, this means that firms share similar suppliers, markets, technology, size and ownership status, what makes it easier for information to circulate across companies. Strong heterogeneity along the mentioned features leads to a more fragmented market where the sharing of information across clusters of firms will be harder. Below, when analyzing the impact of the monetary shock on the trajectory of inflation for different values of the parameter under evaluation, one should keep this reasoning in mind.

Figure 2 represents three trajectories for the inflation rate, one for each value of δ. In the presented cases, there is an initial phase of sharp increase in the value of the inflation rate followed by a decline towards the new steady-state. As expected, the more localized the dissemination of information is, the slower will be the adjustment process in the direction of the new equilibrium. Observe, as well, that a faster convergence is accompanied by a stronger increase in the inflation rate over the transient phase (it reaches a maximum that is as much higher as the less localized is the dissemination of information). Therefore, the extent in which firms in different clusters communicate involves a trade-off: a closer contact turns the convergence faster but simultaneously more abrupt, in the sense that the inflation rate reaches higher values along the transition process.

Figure 2. Inflation trajectory under different degrees of information diffusion across locations



The overshooting of inflation in response to the monetary shock is not an unconventional outcome; it is common to other analyses of the impact of monetary disturbances in the context of sluggish information dissemination, as it is the case of Mankiw and Reis (2003). These authors claim, referring to the reaction of the inflation trajectory to a monetary shock, that 'It overshoots the new lower level briefly as agents learn of the new regime and correct their previous mistakes.' (page 71). Thus, firms begin by perceiving that money growth has left the steady-state in which it used to rest before learning the new equilibrium; as they become acquainted with the new steady-state inflation rate they will, on the aggregate, change their price setting behavior making it possible the convergence to the inflation rate that corresponds to the after-shock rate of money growth.

The overshooting is stronger when information spreads faster across clusters because, in this case, there is a larger number of firms rapidly gaining knowledge on the fact that money supply is growing faster and, thus, prices should grow faster as well. This effect is reversed in a shorter period of time, compared with the other cases, because firms also learn faster the new equilibrium given the central piece of the analysis, i.e., the easiness with which information spreads throughout the space of firms, given a pre-specified pattern of interaction.

6. Conclusion

This note has proposed a framework to analyze the impact of a policy shock over the inflation rate when firms are heterogeneous at a specific level: they are organized in clusters within which information circulates fast; between clusters, the dissemination of information is slower and determined by the distance among them.

Although information may be slow to disseminate across clusters of firms, in the long-run the impact of the assumed shock will be common knowledge for the whole universe of productive units. This implies that all firms will end up adjusting their expectations on prices to the new monetary conditions and, thus, a new stable steady-state will be formed, where inflation equals the after-shock money growth rate.

A numerical example illustrates the process of adjustment. The less localized the dissemination of information is, the faster will be the convergence to the new steady-state; however, this occurs at a cost: a transient phase where the inflation rate deviates (overshoots) more intensely from the corresponding equilibrium level.

Acknowledgments. I would like to acknowledge the helpful comments of two anonymous referees. The usual disclaimer applies.

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Appendix – Derivation of equation (5)

Recover the expression giving the future price level as a weighted average of expected prices,

$$p_{t+1}=\frac{\sum_{\ell=1}^{4m}G_t(1,\ell)}{4m}E_t^I p_{t+1}^*+\frac{\sum_{\ell=1}^{4m}G_t(0,\ell)}{4m}E_t^{II} p_{t+1}^* \qquad (a1)$$

and replace in it the two expectation rules, $E_t^I p_{t+1}^*=p_{t+1}^*$ and $E_t^{II} p_{t+1}^*=p_t+\Delta m|_{\underline{\Omega}}$, and the expression for the desired price at $t+1$, $p_{t+1}^*=p_{t+1}+\alpha y_{t+1}$. This yields

$$p_{t+1}=\frac{\sum_{\ell=1}^{4m}G_t(1,\ell)}{4m}(p_{t+1}+\alpha y_{t+1})+\frac{\sum_{\ell=1}^{4m}G_t(0,\ell)}{4m}\left(p_t+\Delta m|_{\underline{\Omega}}\right) \qquad (a2)$$

Taking $\pi_t := p_t-p_{t-1}$ and rearranging equation (a2), one arrives to the expression of the Phillips curve in the paper,

$$\pi_{t+1}=\alpha\frac{\sum_{\ell=1}^{4m}G_t(1,\ell)}{\sum_{\ell=1}^{4m}G_t(0,\ell)}y_{t+1}+\Delta m|_{\underline{\Omega}} \qquad (a3)$$

Given the definition of money supply, $m_t=p_t+y_t$, one represents equation (a3) as

$$\pi_{t+1}=\alpha\frac{\sum_{\ell=1}^{4m}G_t(1,\ell)}{\sum_{\ell=1}^{4m}G_t(0,\ell)}(m_{t+1}-p_{t+1})+\Delta m|_{\underline{\Omega}} \qquad (a4)$$

Solving equation (a4) with respect to the difference between money and prices, it comes

$$m_{t+1}-p_{t+1}=\frac{1}{\alpha}\frac{\sum_{\ell=1}^{4m}G_t(0,\ell)}{\sum_{\ell=1}^{4m}G_t(1,\ell)}\left(\pi_{t+1}-\Delta m|_{\underline{\Omega}}\right) \qquad (a5)$$

Next, we apply first differences to (a5) and obtain

$$\begin{aligned}&m_{t+1}-p_{t+1}-(m_t-p_t)\\&=\frac{1}{\alpha}\left[\frac{\sum_{\ell=1}^{4m}G_t(0,\ell)}{\sum_{\ell=1}^{4m}G_t(1,\ell)}\left(\pi_{t+1}-\Delta m|_{\underline{\Omega}}\right)-\frac{\sum_{\ell=1}^{4m}G_{t-1}(0,\ell)}{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)}\left(\pi_t-\Delta m|_{\underline{\Omega}}\right)\right]\end{aligned} \qquad (a6)$$

The difference $m_{t+1}-m_t$ in equation (a6) represents the change on money supply after the shock, i.e., $\Delta m|_{\overline{\Omega}}$. Thus, equation (a6) is equivalent to

$$\begin{aligned}\Delta m|_{\overline{\Omega}}-\pi_{t+1}=&\frac{1}{\alpha}\frac{\sum_{\ell=1}^{4m}G_t(0,\ell)}{\sum_{\ell=1}^{4m}G_t(1,\ell)}\pi_{t+1}-\frac{1}{\alpha}\frac{\sum_{\ell=1}^{4m}G_{t-1}(0,\ell)}{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)}\pi_t\\&-\frac{1}{\alpha}\left[\frac{\sum_{\ell=1}^{4m}G_t(0,\ell)}{\sum_{\ell=1}^{4m}G_t(1,\ell)}-\frac{\sum_{\ell=1}^{4m}G_{t-1}(0,\ell)}{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)}\right]\Delta m|_{\underline{\Omega}}\end{aligned}\qquad(a7)$$

Rearranging further,

$$\begin{aligned}&\left[\frac{1}{\alpha}\frac{\sum_{\ell=1}^{4m}G_t(0,\ell)}{\sum_{\ell=1}^{4m}G_t(1,\ell)}+1\right]\pi_{t+1}\\&=\frac{1}{\alpha}\frac{\sum_{\ell=1}^{4m}G_{t-1}(0,\ell)}{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)}\pi_t+\Delta m|_{\overline{\Omega}}\\&+\frac{1}{\alpha}\left[\frac{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)\sum_{\ell=1}^{4m}G_t(0,\ell)-\sum_{\ell=1}^{4m}G_{t-1}(0,\ell)\sum_{\ell=1}^{4m}G_t(1,\ell)}{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)\sum_{\ell=1}^{4m}G_t(1,\ell)}\right]\Delta m|_{\underline{\Omega}}\end{aligned}\qquad(a8)$$

Isolating π_{t+1} in the l.h.s.,

$$\begin{aligned}\pi_{t+1}=&\frac{\alpha\sum_{\ell=1}^{4m}G_t(1,\ell)}{\sum_{\ell=1}^{4m}G_t(0,\ell)+\alpha\sum_{\ell=1}^{4m}G_t(1,\ell)}\left[\frac{1}{\alpha}\frac{\sum_{\ell=1}^{4m}G_{t-1}(0,\ell)}{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)}\pi_t+\Delta m|_{\overline{\Omega}}\right.\\&\left.+\frac{1}{\alpha}\frac{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)\sum_{\ell=1}^{4m}G_t(0,\ell)-\sum_{\ell=1}^{4m}G_{t-1}(0,\ell)\sum_{\ell=1}^{4m}G_t(1,\ell)}{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)\sum_{\ell=1}^{4m}G_t(1,\ell)}\Delta m|_{\underline{\Omega}}\right]\end{aligned}\qquad(a9)$$

and multiplying the numerator of the first fraction in the r.h.s. by the terms inside brackets,

$$\begin{aligned}\pi_{t+1}=&\frac{1}{\sum_{\ell=1}^{4m}G_t(0,\ell)+\alpha\sum_{\ell=1}^{4m}G_t(1,\ell)}\\&\left[\frac{\sum_{\ell=1}^{4m}G_{t-1}(0,\ell)\sum_{\ell=1}^{4m}G_t(1,\ell)}{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)}\pi_t+\alpha\sum_{\ell=1}^{4m}G_t(1,\ell)\Delta m|_{\overline{\Omega}}\right.\\&\left.+\frac{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)\sum_{\ell=1}^{4m}G_t(0,\ell)-\sum_{\ell=1}^{4m}G_{t-1}(0,\ell)\sum_{\ell=1}^{4m}G_t(1,\ell)}{\sum_{\ell=1}^{4m}G_{t-1}(1,\ell)}\Delta m|_{\underline{\Omega}}\right]\end{aligned}\qquad(a10)$$

Now, observe that the numerator of the last fraction in the above equation is such that

$$\begin{aligned}
&\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)\sum_{\ell=1}^{4m} G_t(0,\ell)-\sum_{\ell=1}^{4m} G_{t-1}(0,\ell)\sum_{\ell=1}^{4m} G_t(1,\ell)\\
&=\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)\left[4m-\sum_{\ell=1}^{4m} G_t(1,\ell)\right]-\left[4m-\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)\right]\sum_{\ell=1}^{4m} G_t(1,\ell)\\
&=4m\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)-\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)\sum_{\ell=1}^{4m} G_t(1,\ell)-4m\sum_{\ell=1}^{4m} G_t(1,\ell)+\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)\sum_{\ell=1}^{4m} G_t(1,\ell)\\
&=4m\left[\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)-\sum_{\ell=1}^{4m} G_t(1,\ell)\right]\\
&=-4m\left[\sum_{\ell=1}^{4m} G_t(1,\ell)-\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)\right]
\end{aligned} \qquad (a11)$$

Replacing the last value back into the dynamic equation, one arrives to

$$\begin{aligned}
\pi_{t+1}=\frac{1}{\sum_{\ell=1}^{4m} G_t(0,\ell)+\alpha\sum_{\ell=1}^{4m} G_t(1,\ell)}&\left[\frac{\sum_{\ell=1}^{4m} G_{t-1}(0,\ell)\sum_{\ell=1}^{4m} G_t(1,\ell)}{\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)}\pi_t\right.\\
&\left.+\alpha\sum_{\ell=1}^{4m} G_t(1,\ell)\Delta m\big|_{\overline{\Omega}}-4m\frac{\sum_{\ell=1}^{4m} G_t(1,\ell)-\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)}{\sum_{\ell=1}^{4m} G_{t-1}(1,\ell)}\Delta m\big|_{\underline{\Omega}}\right]
\end{aligned} \qquad (a12)$$

which is equation (5) in the paper.