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# Geological folds in three dimensions

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**Abstract:** The majority of terms that structural geologists use to describe the geometrical features of folds are defined in terms of simple cylindrical fold model. These terms have served well in the past where folds are observed at surface outcrop, and where three-dimensional information is limited. However, new mapping methods, e.g. high precision GPS, seismic and LIDAR deliver more complete 3D data on folded surfaces and reveal that the cylindrical model is an inadequate model of fold geometry. This review examines how existing terms such as the antiform, synform, hinge line and fold inflection line can be re-defined in order to make them useful for describing general non-cylindrical folds.

**Keywords:** fold patterns, principal curvatures, differential geometry, ridge lines.

Current terminology for describing the form of geological folds, e.g. Fleuty (1964), is strongly biased towards structures that are pseudo two-dimensional. It has for several decades provided a practical framework for the analysis of folds seen at surface outcrops, where data collection is limited to exposure surfaces. However, with the introduction of new mapping methods at the surface (e.g. GPS surveying, laser scanning) and in the subsurface (e.g. 3D seismic), a new wealth of 3D data is being delivered from folded geological surfaces. The availability of more complete data for defining fold geometry and folding patterns offers the potential capability for extracting more information on fold evolution and mechanics. It is therefore timely to review the extent to which existing terms are adequate for dealing with 3D structures. This paper will attempt such a review.

## Cylindrical folds

Geologists have long been aware of the complex geometry of some folded surfaces, especially in rocks subjected to multiple folding events. In spite of this, they have made widespread use of simple geometrical models for the description of natural folds. Of these, the cylindrical model is the most popular. A cylindrically folded surface is one that can be swept out by moving a straight line 'generator' parallel to itself in space. The model is essentially two-dimensional because serial sections yield identical fold shapes. A complete description of a cylindrical fold is provided by knowledge of the fold axis, i.e. the orientation of the generator, and the fold's cross-sectional form on the so-called profile plane, i.e. a plane perpendicular to the fold axis. The folded surface has no component of curvature in the direction of the fold axis.

The simplicity of this model lies in the fact that key points observed on the profile plane can be projected in the direction of the fold axis. For example, the fold's point of greatest curvature, highest point and inflection points all project to produce straight lines, the hinge line, crest line and inflection lines respectively.

The attractiveness of the model lies in the fact that cross-sections are easy to construct. Geometrical features observed at any level can be simply projected along the fold axis onto the chosen plane of section. In addition, data from cylindrically folded surfaces are easy to analyze. For example, the fold axis is easy to determine since normals to the surface are co-planar and perpendicular to the fold axis.

On the other hand, there are major limitations of the model. The geometry of the cylindrically folded surface is severely restricted; contains no points of dual curvature and has hinge lines that are straight. Since inflection lines are always parallel to the fold axis, individual folds form parallel strips of infinite length.

Structural geologists, aware of these limitations have classified folds into cylindrical and non-cylindrical types (e.g. Turner and Weiss, 1963; Ramsay, 1967).

It is has been practice to analyze complex non-cylindrical folded areas by subdividing the region into smaller regions of greater simplicity. This is based on the concept of the homogeneous domain; that folded geometries are not self-similar but tend to be more cylindrical on smaller scales. This approach has met with success in many areas and is supported by some laboratory folding experiments discussed below.

**Developable folds**

Recently, structural geologists have considered the developable surface model has an alternative to the cylindrical one (Lisle, 1992; Lisle and Fernández-Martínez, 2005; Thibert *et al.*, 2005; Fernández-Martínez and Lisle, 2009). The developable fold model is less restrictive than the cylindrical model. Like the cylindrical fold, the form of the surface can be swept out by a straight line generator, but now the movement of the generator is less constrained. Successive orientations of the generator no longer need to be parallel but have to mutually intersect (e.g. point P in Fig. 1). These intersection points collectively define a space curve, whose tangent lines constitute the generators of the surface.

In a developable fold, the orientation of the surface remains constant along any given generator. Conical and cylindrical folds are specific cases of developable folds. At all points on a developable surface there is only one principal curvature; the other principal curvature is zero. Hence, the surface is made up entirely of points with zero Gaussian curvature. The principal curvature of zero value is always aligned with the generator (Fig. 1).

In the general case, serial cross-sections are not necessarily geometrically similar. The curvature of the surface increases along the generators in the direction of their convergence (Fig. 1). Inflection lines, which can be used to define the boundaries of adja-

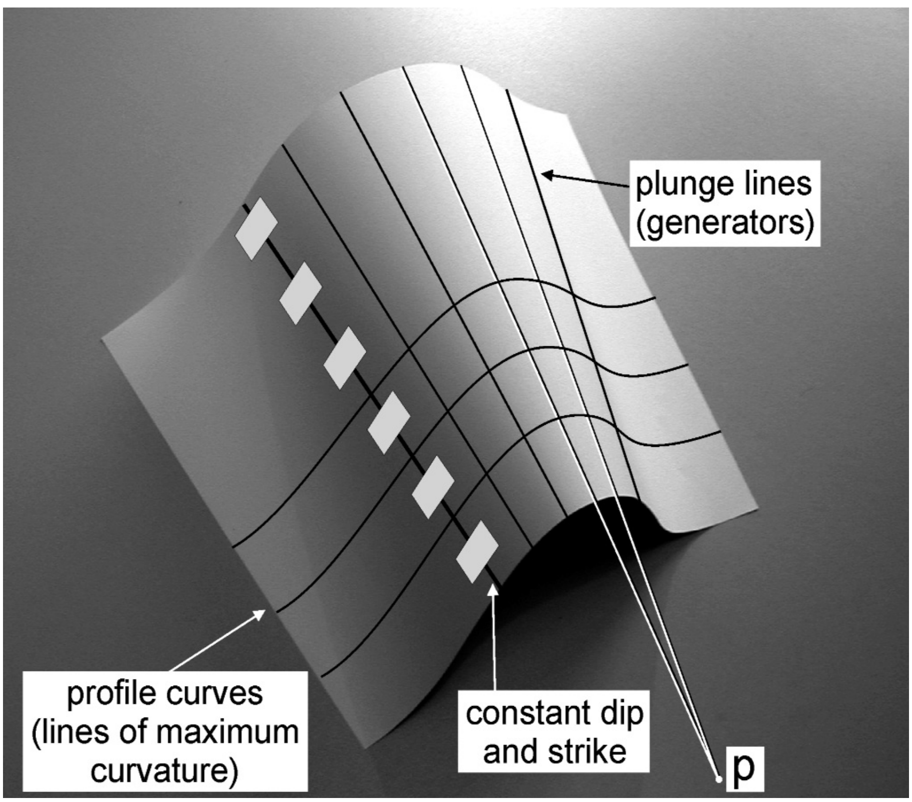


**Figure 1.** Developable fold.

| | cylindrical | developable | general |
|---|---|---|---|
| Fold boundary | ✓ | ✓ | ✓ |
| profile plane | ✓ | × | × |
| hinge line | ✓ | × | × |
| fold axis | ✓ | ✓ | × |
| crest, trough | ✓ | × | × |
| limb | ✓ | × | × |

**Table 1.** Existence of definitions of terms relating to different fold types.

cent folds, coincide with generators (Patrikalakis and Maekawa, 2002). These generators are the tangents of the space curve drawn at points where the curve has no torsion. Terms such as hinge line, crest line and trough line have not been defined in relation to developable folds (Table 1). The lack of a natural profile plane for this type of fold poses a problem in this respect.

Data from developable folds require special treatment. For example, poles to bedding plot on a single curve on the stereogram, though not necessarily on a great or small circle (which characterizes cylindrical and conical folds respectively).

The developable surface is the favoured model for geological folds which undergo no change of bed length (Lisle, 1992). In fact, according to Gauss's *Theorema Egregium* these are the only folds shapes that can form from initially planar surfaces. However, not all developable folds need to be constant bed length, so there is a need to distinguish cases where only isometric bending is involved as *isometric developables*. These latter folds are amenable to restoration; they can be restored by simply rolling out to a flat sheet. However, such restorations need to be performed in three dimensions, since the deformation does accord with plane strain. For a general developable, the plane curve produced by sectioning the surface does not correspond to a plane curve in the undeformed state. Calculating the pre-folding orientation of directional data (palaeomagnetic, current direction) is feasible (Fig. 2).

Developable fold shape is the favoured geometry in the folding of unstretching sheets, e.g. paper. In a geological context this may arise in the deformation of competent units that are thin in relation to their radius of curvature. Where displacement boundary conditions are complex, such as in refolded layers, simple developable folds may not successfully accommodate the required strains. As in crumpled sheets of paper, the isometric bending mechanism may be violated at points of stress/strain localization (Cerda and Mahedevan, 2005). The physical models of Ghosh and Ramberg (1968) illustrate this effect, as do the natural fold patterns of Aller and Gallastegui (1995).

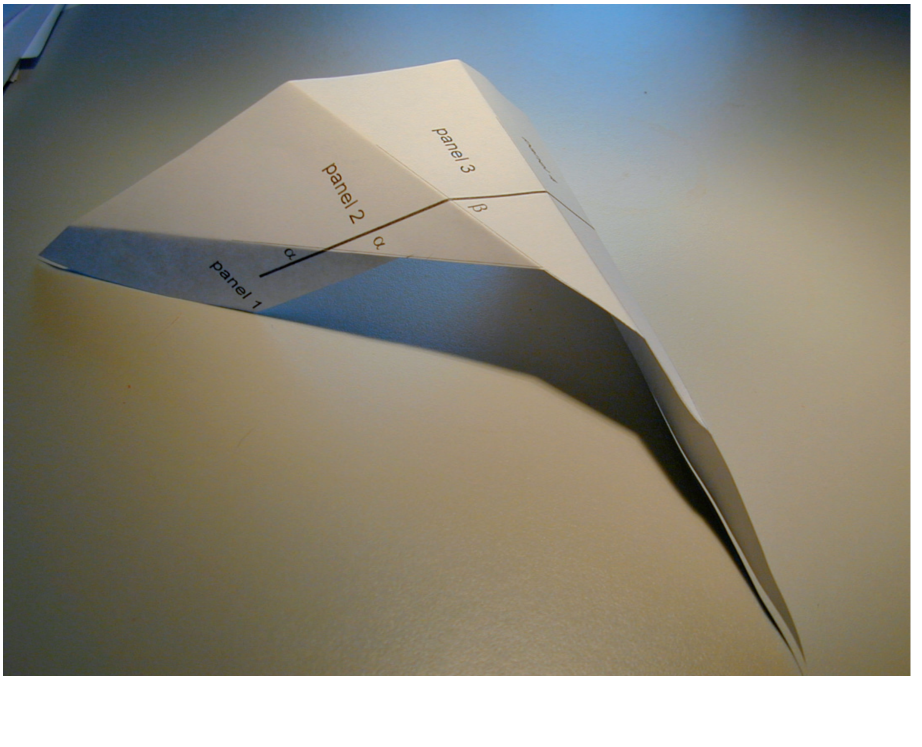


**Figure 2.** Re-orientation of linear structures in an isometric developable fold.

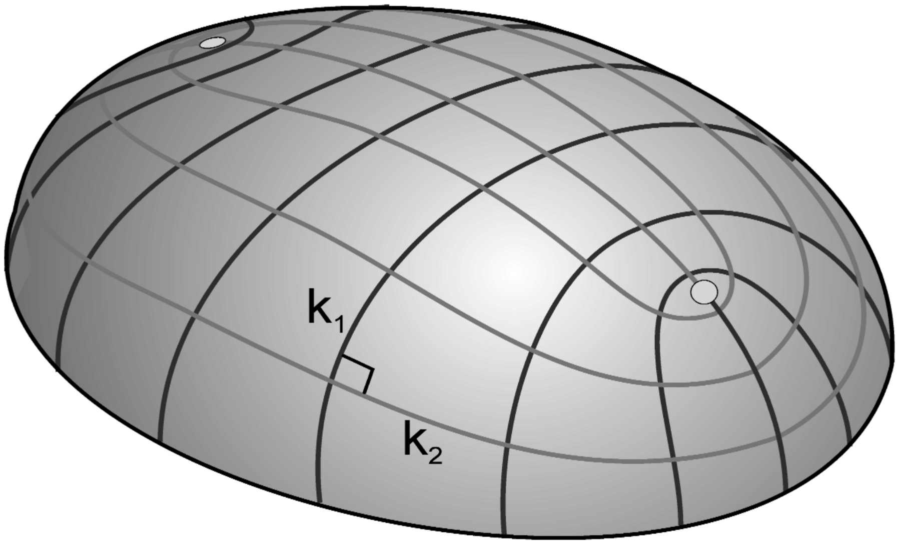


**Figure 3.** Ellipsoidal dome with lines of curvature and two umbilical points (modified from Hilbert and Cohn-Vossen, 1952).

## General folded surfaces

Developable surfaces have specific geometrical properties, and many folds show features that are at odds with those properties. They show structural closure, in which the structure contours describe closed oval forms. These contour patterns are illustrations of Dupin's indicatrix, a figure that results from slicing through a surface on a plane which is parallel to, but slightly lower than, the tangent plane. Its shape relates to the relative magnitudes of the principal curvatures. These oval patterns do not arise where the Gaussian curvature is zero.

To analyze non-developable fold shapes requires new definitions of concepts of hinge line, fold limb, crest line, etc. Workable definitions are still awaited (Table 1).

Studying the arrangement of folds is only possible after a method is found to locate the boundaries of individual folds. Lisle and Toimil (2007) and Mynatt *et al.* (2007) have proposed using the sign of mean curvature (the average of the two principal curvatures) for distinguishing antiforms and synforms. These folds are then divided into synclastic and anticlastic types using the sign of the Gaussian curvature.

A suitable equivalent for the fold axis is difficult to define for an entire fold. The nearest approximation to this is given by the direction of minimum principal curvature (disregarding the sign), though this property will always vary across a single fold, and becomes very unstable in regions where the mean curvature is close to zero.

Lines of curvature, continuous curves tracking the principal curvature directions across the surface, can be determined analytically for some simple surfaces, e.g. the ellipsoid (Hilbert and Cohn-Vossen, 1952). In principle, this orthogonal network provides a useful reference frame for defining fold features (Fig. 3). However, such lines are difficult to construct for real geological surfaces. They form swerving patterns in the close proximity of so-called umbilical points. These are points where the two principal curvatures are equal in magnitude.

## Conclusions

i) At present, the geometrical analysis of fold patterns, e.g. en echelon fold arrays, is hindered by the lack of a sound terminological framework capable of treating non-cylindrical folds, ii) in their search for suitable terms for describing folds, structural geologists could take advantage of developments in the field of computer aided design, where similar problems are being tackled.

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