Trabajos de Geología, Universidad de Oviedo, 29 : 368-371 (2009)

Limits and biases on the three-dimensional vorticity analysis using porphyroblast system: a discussion and application to natural example

D. Iacopini[1, 2*], C. Frassi[1], R. Carosi[1] and C. Montomoli[1]

[1]*Dipartimento di Scienze della Terra, Università degli studi di Pisa, 56100, Pisa, Italy.*

[2]*Now at: Geology and Petroleum Geology Department, University of Aberdeen, AB24 3FX, UK.*

[*]*e-mail: d.iacopini@abdn.ac.uk*

Abstract: A description on the systematic errors associated with the measurement of the vorticity numbers is presented and an application to a real example is discussed. We show that strong biases and systematic errors derive both from some physical assumptions (Jeffery or Mulchrone model) rarely encountered by natural systems as well as by the fact that these vorticity techniques require measurements of the geometrical parameters of porphyroblasts-matrix system by using outcrop surfaces or thin section methods that are inherently biased. Applying different vorticity plots we analyse an example and discuss in detail the effective systematic errors distribution and the ambiguity in measuring the mean vorticity numbers Wn.

Keywords: vorticity, porphyroclast, shear zones, errors distribution.

Three main analytical techniques (Passchier, 1987; Wallis, 1995; Holcombe and Little, 2001) using rigid porphyroclast to estimate Wn, and based on the theory proposed by Jeffery (1922), are commonly employed to characterize flow within shear zones in a variety of tectonic settings (see Jessup *et al.*, 2007 and reference therein). All these techniques rely on a fundamental relationship between Wm (main vorticity number), the shape factor R and the angle of porphyroclast long axis with respect to the foliation or stretching direction to define a threshold number Rc below which they continuously rotate and above which they record a stable sink position. However, the great majority of these vorticity analyses performed within shear zones totally ignored the real systematic errors biases linked to such measurements, often assuming the vorticity numbers obtained were not affected by measurement errors. This could give rise to results without a clear physical and geological validity. To our concern, this main limit is primary due to the fact that few detailed analysis of the effective sources of these errors and few self critical discussion on the limit of the measurement techniques employed has been tempted so far (Forte and Bailey, 2007; Mulchrone, 2007b; Iacopini *et al.*, 2008). Moreover, all these techniques are derived from the Jeffery solution that assumes the rigid porphyroclast is immersed in Newtonian fluid with no slip at the boundary. This physical assumption has been adopted as *a priori* condition but needs to be proved or tested. This now become partly possible thanks to Mulchrone (2007a), who published a series of plot showing the relationship between mean kinematics vorticity number Wm, the shape factor R and the angle η.

In this contribution, we show theoretically and by mean of a natural example, that both in the recognition of the vorticity vectors orientation and in the measurement of the porphyroclast aspect ratio, two main systematic errors have to be encountered, especially if the strain intensity registered by the deformed rocks is not very high. Finally, within the data set obtained from vorticity analysis, we perform a best fit using both the Passchier and Mulchrone curves and try to investigate if the porphyroclast behaved perfectly coupled with the matrix during the deformation events.

Systematic source errors

The kinematic vorticity number Wk has its origin in fluid dynamics and records the amount of rotation relative to the amount of stretching at a point in space and in an instant in time. It has been introduced into geological literature because it represents basic flow parameters able to describe flow kinematics, e.g. to distinguish between pure and simple shear within shear zones. Assuming a steady state deformation, its application into geology has been facilitated by the use of Mohr circle strain that permits to efficiently correlate Wk to the velocity gradient tensor and the deformation matrix. To estimate Wk, several methods called vorticity gauges, based on different microstructures have been proposed and are currently under development. The method we will re-analyse is based on rotational behaviour of a rigid object within non-coaxial flow.

One method is the technique proposed by Passchier (1987), and based on the following relation:

$$\eta = \frac{1}{2}\sin^{-1}\frac{Wk}{R^*}\left(\sqrt{1-Wk^2}-\sqrt{R^{*2}-Wk^2}\right) \qquad (1),$$

where the angle η between the maximum axis of an irrotational rigid object in the XZ finite plane and the extensional eigenvector (materialized by the straight domain of the tails) is a function of Wk and the critical aspect ratio R* (B* in Passchier, 1987).

Another technique is the one proposed by Holcombe and Little (2001) based on the following relationship:

$$\dot{\theta} = \left[\frac{\dot{\gamma}R^2\sin^2\theta+\dot{\gamma}\cos^2\theta-(R^2-1)\dot{\varepsilon}\sin 2\theta}{R^2+1}\right] \qquad (2),$$

where R is the aspect ratio, while $\dot{\gamma}$ and $\dot{\varepsilon}$ are the far field bulk simple shear and the pure shear respectively.

Both methods use the geometry, aspect ratio of mantled porphyroclasts as well as the inclusion trails geometry of porphyroblasts to determine the amount of vorticity. In practice, the methods consist of compiling diagrams showing the orientation of the long axis of porphyroblasts vs. the recrystallized tails direction (eq. 1) or vs. the orientation of the internal foliation (eq. 2). Both of them are measured in thin section, cut parallel to the stretching lineation and orthogonally to the main foliation. These data are then compared to theoretical curves obtained using equations (1) and (2). From the two equations at least two source errors could be recognized:

1) Source error due to the fact that the sectional radii of a porphyroclast measured in thin section misrepresent the true maximum sectional vorticity plane (orthogonal to the vorticity vector) as well as the true porphyroclast radii in a sample (Fig. 1).

As predicted by the analytical solution (Passchier, 1987; Mulchrone, 2007a) this effect is mainly due to the fact that porphyroclast axes during the shearing show also a revolutional movement and, as a consequence, the porphyroclasts are always slightly disoriented with respect to the mean stretching direction assumed as a reference direction. As a result, vorticity analysis is performed in sections that are not always perfectly orthogonal to the vorticity vector of the porphyroclast system. As showed in figure 1, apparent inclusion trail profiles and aspect ratios are obtained from non-central sections and this implies that in

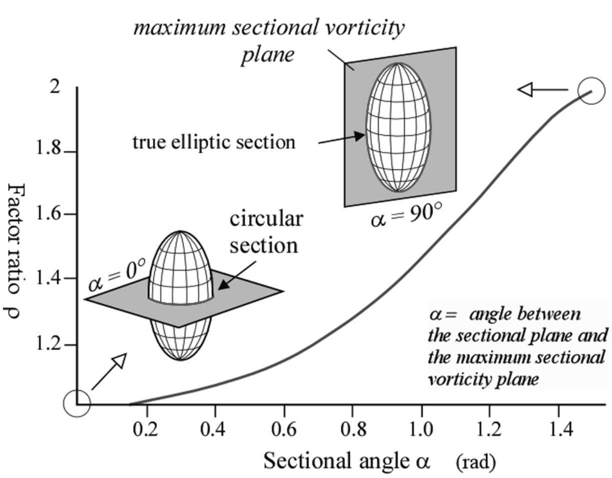

Figure1. Diagram showing the effective radius measured along different misoriented plane to the main sectional vorticity plane. The porphyroclast is idealized as a biaxial ellipsoid. ϕ represents the angle between the porphyroclast main axes and the sectional cutting plane. Section with ϕ = 0 represents a plane orthogonal to the main axis. Section with ϕ = 90 represents a plane parallel to the main axes and measures the true main porphyroclasts axes.

both methods a systematic error is introduced. For example, assuming the porphyroclast is a biaxial ellipsoid, figure 1 shows that if we cut the porphyroclast

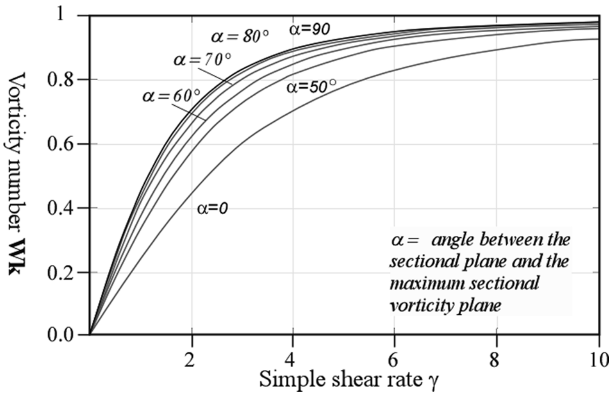

Figure 2. Diagram showing the distribution of kinematic vorticity as a function of simple shear component of flow calculated along different planes with different orientations to the maximum sectional vorticity plane. Black line indicates kinematic vorticity calculated along the maximum sectional vorticity plane. Gray lines indicate kinematic vorticity for sectional planes respectively at 5, 10, 15, 20 and 25° from the maximum sectional vorticity plane.

with a misfit of 0-20º we can obtain an error of 0-0.4 in the estimation of the aspect ratio. The main effect in estimating the vorticity values using Passchier plot (Passchier, 1987) are shown in figures 2 and 3: it is shown that we can expect an error of 0.2 in the estimation of the vorticity errors (Fig. 2). If we know independently the shear component and use a plot Wm vs. γ, a similar error is expected again if we are not exactly in the sectional vorticity plane (Fig. 3). Figure 3 shows that for high shear strain components (Wm~1) the curves become all closer each other decreasing the effective error. In all cases, the more the shear zones are affected by high shear strain, the more the porphyroclasts are well oriented along the stretching direction and the lower the error expected.

2) A second systematic error derives from the fact that the analyses are usually performed in two dimensions (Tikoff and Fossen, 1995). In this case, there is always an underestimation of the orthogonal possible stretching direction effect. If such component is not zero, it induces an overestimation of the real vorticity number. This systematic error has been estimated by Tikoff and Fossen (1995) to be at least 0.05 if sections are perfectly orthogonal to the vorticity vector.

Other possible errors could derive from an underestimation of the dilatancy effectively registered by the shear zones. In this case, for high strain shear zones as showed by figure 4 its effect is far more lower respect the previous described source errors but could not reasonably be neglected for low simple shear values.

Summarizing, if we are dealing with shear zones characterized by general shear with vorticity numbers Wm ≤ 0.9-1.0, systematic errors should be taken into account and a biases of, at least 0.2 or 0.3 could be expected especially if porphyroclast are not perfectly iso-oriented to the main stretching directions. This implies that vorticity numbers strongly affected by pure shear component, e.g. transpressive shear zones, could not be well estimated and careful sampling as well as knowledge of mis-fitting angle are needed.

Consideration on the slipping effect

Another type of systematic error comes out from the main physical assumptions of the various vorticity analysis techniques taken into account in this contribution: the rigid porphyroclasts-matrix interface does not have any slipping component and the shear zones are not confined. *A posteriori*, in the vast majority of

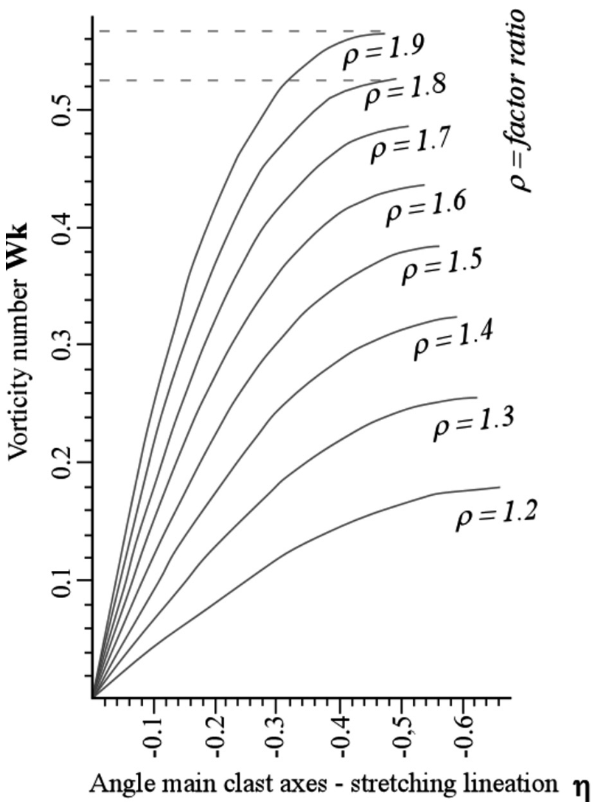

Figure 3. Diagram showing how different aspect ratios could produce different vorticity plot curves with different critical vorticity number Rc. η represents the orientation of the main rigid clast axes with respect to the stretching direction. R is the aspect ratio of the rigid clast.

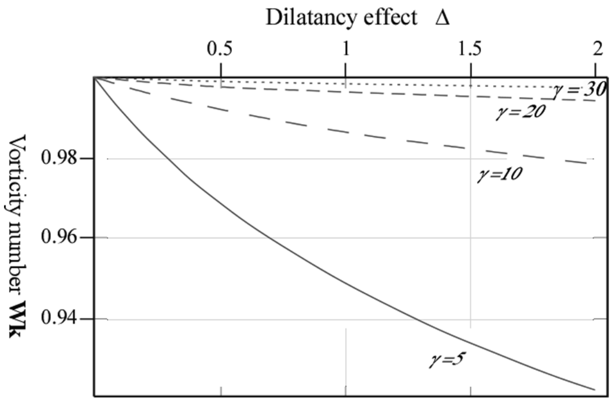

Figure 4. Diagram showing the dilatancy effect on the vorticity numbers, at various fixed shear components. For high shear values (1.5 to 2), the effect of dilatancy on vorticity numbers is very low and could be neglected. For low shear strain values the effect becomes important.

vorticity analyses, when we perform a kinematic analysis of shear zones, the real effectiveness of these two properties cannot be easily tested or demonstrated. If during the shearing event the interface behaved with a slipping component, the Jeffery model is invalidated, and as shown by Pennacchioni *et al.* (2001) and Mulchrone (2007b), we should expect a different relationship between object inclination and the vorticity of flow respect to the one described in equations (1) and (2). As a consequence, when we analyse a population of porphyroclasts using a η vs. R plot diagram or a η vs. plots as a blind application of the classical curves (Passchier, 1987; Jessup *et al.*, 2007), the best fitting approach could produce erroneous results especially if the data are quite scattered. The best thing would be to test which of the slipping or no slipping related curves better approximate the distribution of points and then try to understand which systematic errors described above should be taken in account. As suggested by Passchier (1987) and Mulchrone (2007a) to obtain realistic estimate of vorticity numbers and to be able to distinguish the ability of the different curves in fitting the distribution of data in a η vs. R plot, a large number of porphyroclast are required.

Conclusion

A discussion on the possible systematic errors distribution expected in performing vorticity analyses is presented. From theoretical consideration we expect to obtain some ambiguities in vorticity numbers measurements especially if shear zones registered transpressive or pure shear dominated deformation. These ambiguities are mainly due to the errors induced during the sectional radii estimation and the recognition of the maximum vorticity plane. Finally, during the analysis of the η vs. R plot point data set we suggest to apply the all-slipping and non-slipping vorticity plots in order to discriminate the potential slipping effect in the kinematics history of the high strain shear zones investigated. In case of scattered η vs. R data sets, we suggest to apply the all-slipping and non-slipping vorticity plots as best fitting curves. This will fine tune the vorticity analysis and enable to check likely slipping effect in the porphyroclast-matrix system.

References

Forte, A. M. and Bailey, C. M. (2007): Testing the utility of porphyroclast hyperbolic distribution method of kinematics vorticity analysis. *J. Struct. Geol.*, 29: 983-1001.

Holcombe, R. and Little, T. (2001): A sensitive vorticity gauge using rotated porphyroblasts and it s application to rocks adjacent to Alpine fault, New Zealand, *J. Struct. Geol.*, 23: 979-989.

Iacopini, D., Carosi, R., Montomoli, C. and Passchier, C. W. (2008): Strain analysis of flow in the Northern Sardinian Variscan Belt: Recognition of a partitioned oblique deformation event. *Tectonophysics*, 446: 77-96.

Jeffery, G. B. (1922): The motion of ellipsoidal particles in a viscous fluid. *Proc. R. Soc. London, Ser-A*, 102: 161-179.

Jessup, M. J., Law, R. and Frassi, C. (2007): The rigid Grain Net (RGN): An Alternative method for estimating mean kinematics vorticity numbers (Wm). *J. Struct. Geol.*, 29: 411-421.

Mulchrone, K. (2007a): An analytical solution in 2D for the motion of rigid elliptical particles with a slipping interface under a general deformation. *J. Struct. Geol.*, 29: 950-960.

Mulchrone, K. (2007b): Shape fabrics in populations of rigid object in 2D: Estimating finite strain and vorticity. *J. Struct Geol.*, 29: 1558-1570.

Passchier, C. W. (1987): Stable positions of rigid objects in non coaxial flow – a study in vorticity analysis. *J. Struct. Geol.*, 9: 679-690.

Pennacchioni, G., Di Toro, G. and Mancktelow, N. S. (2001): Strain-insensitive preferred orientation of porphyroclasts in Mont Mary mylonites. *J. Struct. Geol.*, 23: 1281-1298.

Tikoff, B. and Fossen, H. (1995): The limitations of 3-dimensional kinematic vorticity analysis. *J. Struct. Geol.*, 17: 1771-1784.

Wallis, S. R. (1995): Vorticity analysis and recognition of ductile extension in the Sanbagawa belt, Japan. *J. Struct. Geol.*, 17: 1077-1093.