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Numerical paleostress analysis – the limits of automation

M. KERNSTOCKOVA[1*] AND R. MELICHAR[1]

[1]*Department of Geological Sciences, Faculty of Science, Masaryk University, Kotlarska 2, 611 37 Brno, Czech Republic.*

[*]*e-mail: mujstock@email.cz*

Abstract: The purpose of this paper is to highlight the main problematic questions limiting the automation of numerical paleostress analysis of heterogeneous fault-slip data sets. It focusses on problems concerning the creation of spurious solutions, the separation of data corresponding to several tectonic phases, and the precision of measurement. It offers some ways of dealing with these problems and trying to control their influence on the accuracy of solutions.

Keywords: numerical paleostress analysis, fault-slip data, principal stresses, reduce stress tensor.

The stress field reconstruction of deformed rocks is a key to understanding tectonic events in studied areas. It is usually based on an analysis of fault-slip data sets. Fault-slip data (e.g. orientation of faults, orientation of striation, sense of slip) code information about the stress state which causes these brittle structures. Changes of stress field can cause new fault formation or the reactivation of older ones. Multiple striations on some fault surfaces indicate this reactivation. Numerical searching for appropriate stress states acting on studied areas is the focus of many studies (e.g. Fry, 1999; Yamaji, 2000; Shan *et al.*, 2004a). Automation of this process is complicated by many factors such as the variability of the stress field in geological time, the precision of measurements, the origination of spurious solutions, etc.

Homogeneous vs. heterogeneous data sets

There are several approaches to determine possible paleostress conditions. The simple inverse method (Carey and Brunier, 1974; Nemcok and Lisle, 1995; Yamaji, 2000; Shan *et al.*, 2004a; among others) can be applied to fault-slip data, which represent a single tectonic phase – it means that all faults were activated under the same stress state. In this case the fault-slip data set is called homogeneous (Angelier *et al.*, 1982). In fact, we use a direct solution to calculate the reduced stress tensor (Angelier *et al.*, 1982) for a four-element fault-slip group. The reduced stress tensor is represented by three directions of principal stresses σ_1, σ_2, σ_3 ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) and its relative value is expressed by a shape parameter (e.g. Lode's ratio μ_L). Otherwise, the stress ellipsoid represents the existing stress state constituted by the nine-dimensional stress tensor vector $\mathbf{T}_\sigma$ (Melichar and Kernstockova, 2009).

If we process a homogeneous data set with more than four faults, the optimal stress tensor is distinguished simply, just like the best-fit solution based on the least square method. Let N be the number of all faults with striation. Each fault-slip datum can be expressed as a nine-dimensional unit vector C (Shan *et al.*, 2004b; Li *et al.*, 2005, Melichar and Kernstockova, 2009). Let $\mathbf{C}_i$ be the i-th vector and δ_i the angle between vector $\mathbf{C}_i$ (for $i = 1...N$) and any other 9D vector (e.g. the normalized stress vector $\mathbf{T}_\sigma$ we are looking for, which must be perpendicular to all $\mathbf{C}_i$ vectors). Let us choose $\cos\delta_i$ as the representative of deviation from the perpendicular position. This choice enables us to find the direction with minimal deviation mathematically. Afterwards, for a homogeneous data set, the optimal stress vector $\mathbf{T}_\sigma$ can be calculated just like a best-fit solution by minimizing the function:

$$\sum_{i}^{n} cos^2\, \delta_i \qquad (1),$$

in which $\cos\delta_i$ can be replaced with the scalar product of unit vectors $\mathbf{C}_i$ and normalized stress vector $\mathbf{T}_\sigma$:

$$\sum_{i}^{N}(\mathbf{T}_\sigma{}^{\mathsf{T}} \cdot \mathbf{C}_i)^2 = \sum_{i}^{N}\mathbf{T}_\sigma{}^{\mathsf{T}} \cdot \mathbf{C}_i \cdot \mathbf{C}_i{}^{\mathsf{T}} \cdot \mathbf{T}_\sigma \qquad (2),$$

where vector $\mathbf{T}_\sigma$ is unknown but constant. Thus equation (2) can be rearranged as:

$$\mathbf{T}_\sigma{}^{\mathsf{T}}\left(\sum_{i}^{N}\mathbf{C}_i \cdot \mathbf{C}_i{}^{\mathsf{T}}\right)\cdot \mathbf{T}_\sigma = \mathbf{T}_\sigma{}^{\mathsf{T}} \cdot [\mathrm{M}] \cdot \mathbf{T}_\sigma \qquad (3).$$

Symbol [M]:

$$[\mathrm{M}] = \sum_{i}^{N}\mathbf{C}_i \cdot \mathbf{C}_i{}^{\mathsf{T}} \qquad (4),$$

denotes the orientation matrix of a fault-slip data set.

This orientation matrix is symmetric; the eigenvector corresponding to the second smallest eigenvalue of this matrix designates a stress vector with a minimal sum of deviations between vectors $\mathbf{C}_i$ and $\mathbf{T}_\sigma$, i.e. the optimal stress vector which is as perpendicular as possible to each of the $\mathbf{C}_i$ vectors. This is the completely 9D parallel to Fry's (1999) procedure in 6D.

However, the stress conditions vary in geological time and thus field data are usually heterogeneous. In this case a multiple inverse method (Yamaji, 2000) is used to process data and estimate possible stress states.

The heterogeneity of a field data set is the basic problem of fault-slip data paleostress analysis. There are some indicators to distinguish the minimal number of stress phases (e.g. multiple striations on a single fault surface) or the relative ages of different stress states, but it is not so easy to assign an individual fault to certain tectonic phases.

A field data set is a mixture of polyphase stress state records on fault surfaces which we must process together. The fault-slip data are combined just into four-element groups and the reduced stress tensor (shape parameter μ_L and orientation of stress ellipsoid) is calculated for each group. Any of the four fault-slip data is a vector $\mathbf{C}$ in 9D space and the stress solution is represented by vector T_σ perpendicular to each of them. It means $\mathbf{C}\cdot\mathbf{T}_\sigma = 0$, where symbol "dot" marks the scalar ("dot") product of two vectors.

A group with four homogeneous fault-slip data (i.e. four fault-slip data corresponding the same tectonic phase) provides the true results which characterize the real paleostress conditions activating the movement along these faults. The stress tensor calculated for a heterogeneous four-fault group is not reliable. Projections of directions calculated from heterogeneous data sets – false results – are dispersed, whereas the true results obtained from homogeneous data are grouped in clusters.

The density maximum indicates some of the possible directions of considered principal stress. The number of such clusters indicates a minimal number of paleostress phases. The best way to recognize the density maxima of the correct solution is the direction density analysis in (9 – 4 =) 5-dimensional space (Melichar and Kernstockova, 2009).

For the density distribution calculation we use the Watson distribution usually used for contour plots, generalized on multidimensional space. Searching for the optimal stress vector $\mathbf{T}_\sigma$ can be graphically expressed by the density distribution $h(\mathbf{a})$ in direction $\mathbf{a}$:

$$h(\mathbf{a}) = \sum_{i} W(\mathbf{a},\mathbf{C}_i) \qquad (5),$$

where $W(\mathbf{a}, \mathbf{C}_i)$ is an effect of $\mathbf{C}_i$th measurement of density in direction $\mathbf{a}$ (Fig. 1):

$$W(\mathbf{a},\mathbf{C}_i) = \frac{1}{k}\cdot\exp[\kappa\cdot(\mathbf{a}\cdot\mathbf{C}_i)^2] \qquad (6),$$

where k is a constant determining that the probability in all directions is equal to one (to compute only the relative concentrations is correct by simply applying $k = 1$); κ is the shape parameter termed the *concentration parameter*, because the larger the value of κ, the more the distribution is concentrated around direction $\mathbf{a}$ (Fisher *et al.*, 1987).

Paleostress analysis – limits of automation

The rate of automation is an important criterion when calling the software "user friendly". However, in many cases it is difficult to transform the problematic task in a numerical way. Usually it is possible to find an algorithm to solve the problem automatically (e.g. Shan, *et al.*, 2004a; Yamaji *et al.*, 2006) but at the expense of the accuracy of solution. In paleostress analysis of heterogeneous fault-slip data sets there are also some difficulties of this kind. To make the problem more graphic we can use the parallelism between paleostress analysis of fault-slip data in 9D and fold axes analysis of bedding planes taken from several different folds in 3D space.

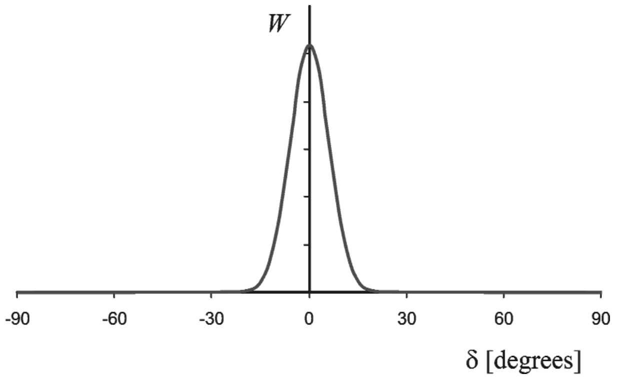

Figure 1. Probability density function of the Watson distribution for κ = 50 demonstrating one measurement effect on density.

Spurious solutions

In the paleostress analysis of a heterogeneous fault-slip data set there is a problem of some analysis outputs being spurious and not reflecting real stress conditions in rock. These solutions are products of stress tensor numerical calculation from heterogeneous four fault-slip data groups.

The analogous situation is well known in fold-axis analysis, where we are looking for the fold β-axis just as a direction perpendicular to the bedding planes unit normal vectors $\mathbf{n}_1$, $\mathbf{n}_2$. Fold β-axis is calculated like a vector product:

$$[\mathbf{n}_1 \times \mathbf{n}_2] = \boldsymbol{\beta} \qquad (7).$$

In the studied area we commonly measure several bedding planes of every fold. In a similar way as two bedding planes ($\mathbf{n}_1$, $\mathbf{n}_2$) of two different folds produce a β-axis of a nonexistent fold, four fault-slip data ($\mathbf{C}_\mathrm{I}$, $\mathbf{C}_\mathrm{II}$, $\mathbf{C}_\mathrm{III}$, $\mathbf{C}_\mathrm{IV}$ as symmetrical components in 5D subspace, see Melichar and Kernstockova, 2009) corresponding to different tectonic phases produce spurious solutions of the stress state $\mathbf{T}_\sigma$:

$$\left[\mathbf{C}_\mathrm{I} \times \mathbf{C}_\mathrm{II} \times \mathbf{C}_\mathrm{III} \times \mathbf{C}_\mathrm{IV}\right] = \mathbf{T}_\sigma \qquad (8).$$

These spurious solutions are dispersed whereas the correct ones are clustered, so they can be simply distinguished by the density distribution function $h(\mathbf{a})$ mentioned above.

The situation is more complicated because of the preferred orientation of field data. Preferred orientation of faults brings spurious density maxima of solutions. Let us demonstrate the problem using the fold-axis analysis parallel.

The analysis of bedding orientation usually leads to several clusters of bedding normals instead of whole “great circles”. Consequently, in the case of a more than one-fold system, the determination of the correct “great circles” is not explicit: some β-axis calculated as a pole to the “great-circle” defined by pairs of clusters may not represent real fold axes (Fig. 2a). Analogously, due to the preferred orientation of faults, only segments of “great hypercircles” are generated. The determination of an appropriate stress state $\mathbf{T}_\sigma$ as a pole to this “great-hypercicle” is at risk of resulting in the same error.

How can this problem be addressed? Spurious maxima could be recognized from the character of fault surfaces (e.g. material of accretion steps, alterations). Faults with different characters separated into one group probably represent a spurious tectonic phase created by the preferred orientation of input data, whereas faults with the same characters separated into one group were probably activated by common real stress.

Numerical separation of fault-slip data corresponding to several tectonic phases

Another problem is related to the fact that some fault-slip data correspond to several phases, so it is difficult to separate them correctly and assign them their corresponding stress state. Analogously, in fold-axis analysis this sorting problem is well known as well. Data from two or more folds bring two or more “great circles” of appropriate bedding poles. Bedding poles surrounding the intersection point of these two or more “great circles” are disputative in terms of their assignment to a certain β-axis represented by the pole of “great circle” (Fig. 2b). Analogously, heterogeneous fault-slip data create points of several “great hypercircles”. The poles of these “great hypercicles” indicate possible stress states, a possible tectonic phase. Consequently, fault-slip data surrounding the intersection of two “great hypercircles” are similarly ambiguous with respect to their assignment to a certain tectonic phase. In practice, this separation can be performed numerically, but this is only an artificial separation. If there are common characters of some tectonic phases, the accuracy of this separation could be verified. Fault-slip data with the same characters of fault surfaces should belong to the same group.

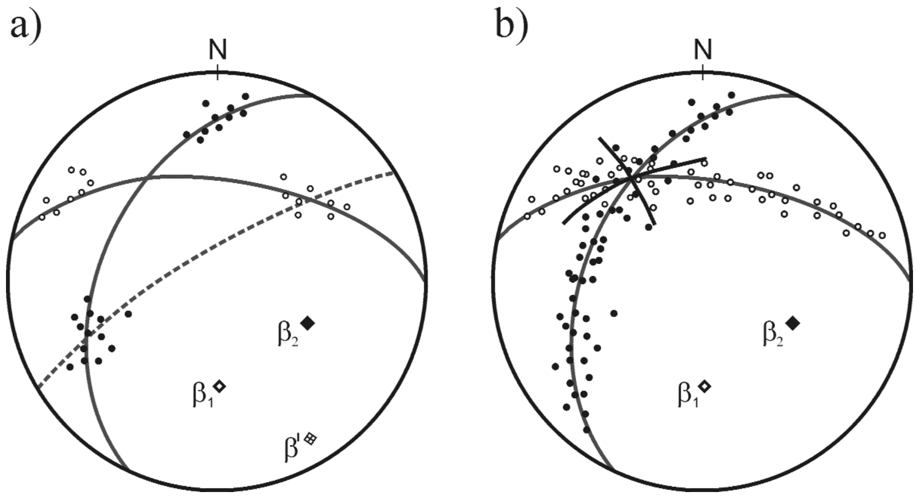

Figure 2. A problem with separation of heterogeneous fault-slip data schematically illustrated on analogous situation with β-axis analysis. Full circles represent bedding normals of fold 1, empty circles denote bedding normals of fold 2. Symbols β_1, β_2 indicate *β*-axes of fold 1 and fold 2. a) Problem of data preferred orientation: the dashed line and *β'* symbol represent one possibility of the false *β*-axis determination, b) problem of numerical separation in the case of data corresponding to more tectonic phases: short intersecting lines illustrate one possible artificial separation producing matching of some bedding normals to incorrect *β*-axis.

Moreover, in the partitioning method there is also the singularity problem. The main condition for separation – the fault-slip data vector **C** is perpendicular to the stress tensor vector $\mathbf{T}_\sigma$ (i.e. $\mathbf{C} \cdot \mathbf{T}_\sigma = 0$) – is always valid for faults perpendicular to any principal stress σ_1, σ_2, σ_3. But in this case, the shearing stress reacting on the fault surface is zero in any direction; thus it is zero in direction **l** and **m** too and the fault could not be ever reactivated (vector **n** is the normal to the fault surface and is down directed, vector **l** is a vector oriented in the direction of hanging-wall movement (i.e. parallel to striation) and the vector **m** lies on the fault surface at a right angle to the striation, which means it is perpendicular to **n** and **l**).

To evaluate the possibility that the fault will be reactivated, the relative Mohr criterion Δτ' based on the estimated angle of internal friction ϕ of the deformed rock, the normalized shear stress $\bar{\tau}$ and the normalized normal stress $\bar{\sigma}$ was proposed:

$$\Delta\tau' = \frac{\bar{\sigma} \cdot \tan\phi - \bar{\tau}}{\sqrt{\tan^2\phi + 1}} \quad (9),$$

where:

$$\bar{\tau} = \frac{2\tau}{\sigma_1 - \sigma_3} \quad (10),$$

and:

$$\bar{\sigma} = \frac{2\sigma_n - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \quad (11),$$

Symbol τ denotes tangential and σ_n normal stresses on the fault surface. The higher the value of Δτ', the higher the probability of reactivation (Fig. 3a).

Precision of measurement

The precision of measurement is an issue for all kinds of analysis. Precision of measurement in fault-slip data analyses controls the precision of solutions. Input data rounded to thousands of degrees provide distinct solutions; solutions of analysis from data rounded to whole degrees are still acceptable but the increasing inaccuracy of input data rapidly diminishes the precision of solutions. The more the fault-slip data are analyzed, the more precise the calculated solutions of the stress states become. Especially if a small amount of fault-slip data is analyzed, the position of measured striation must be tested in accordance with the orientation of the measured fault surface. If striation does not lie on the fault surface and the deviation is small, data should be orthogonalized; if the deviation is large it would be better to exclude the data from the analysis.

Moreover, a stress tensor calculated from nearly parallel data (it means nearly parallel faults with nearly parallel striation) is numerically correct but practically error prone and this solution should be excluded. The scale of parallelism is indicated by the *Gram determinant G* of vectors $\mathbf{C}_{\mathrm{I}}$, $\mathbf{C}_{\mathrm{II}}$, $\mathbf{C}_{\mathrm{III}}$, $\mathbf{C}_{\mathrm{IV}}$, $\mathbf{A}_{xyz}$, $\mathbf{A}_{xy}$, $\mathbf{A}_{xz}$, $\mathbf{A}_{yz}$ (Melichar and Kernstockova, 2009). The magnitude of the square root of the Gram determinant $\sqrt{G}$ near to zero indicates that two or more of the pieces of input data are nearly parallel and this solution should

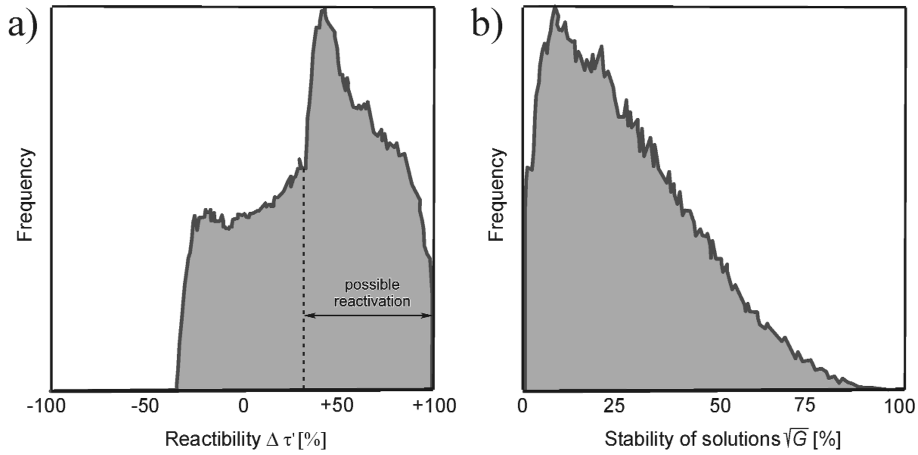

Figure 3. Distribution of the reactivity and stability of solutions of real fault-slip data from the Brno Massif granitoids. a) The best point to limit the area of reactivation is the point where the distribution of reactivity steeply increases, b) visualization of stability distribution enables us to exclude the unstable solutions from the analysis.

be excluded. Otherwise, the more the magnitude of the square root of the Gram determinant approximates to superior limit 1/8 (Melichar and Kernstockova, 2009), the more perpendicular are the vectors $\mathbf{C}_{I}$, $\mathbf{C}_{II}$, $\mathbf{C}_{III}$, $\mathbf{C}_{IV}$ and the more stable is the solution. Consequently, the magnitude of the Gram determinant is a good criterion for controlling the stability of solutions (Fig. 3b).

Conclusions

The automation of the numerical paleostress analysis of heterogeneous fault-slip data has some limits which emerge especially in the effort to separate or match fault-slip data to certain tectonic phases. To deal with these limits some control parameters are proposed (κ, $\Delta\tau'$, $\sqrt{G}$). Therefore, numerical automation of paleostress analysis is practicable but in the charge of a geologist, particularly when the analysis of the results are interpreted.

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