



# Prediction of thrusting sequence based on maximum rock strength and sandbox validation

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**Abstract:** This research aims to develop and validate simple procedures, accounting for mechanical equilibrium and maximum rock strength, to select the dominant mode of folding and to predict sequences of thrusting in fold-and-thrust belts and accretionary wedges. An inverse analysis of sandbox experiments is proposed to validate the procedures, leading to an estimation of the fault strength from the position, and the lifetime of the first thrust. The statistics of experimental uncertainties is constructed to obtain probability distributions of the friction parameters. The procedure will be applied to a real field case: the Agrio fold-and-thrust belt, Neuquén basin, Argentina.

**Keywords:** mechanical equilibrium, maximum rock strength, thrusting sequence, wedge, sandbox experiments, statistical analysis.

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The first objective of this work is to provide predictions of the positions, dips and lifetime of thrusts in accretionary wedge and fold-and-thrust belts. We propose to develop simple procedures, compared to the finite-element method, so that the thousand of tests required for inverse analysis, could be performed efficiently.

Prediction of thrusting sequence has its roots in the stability of a perfect triangular cohesionless wedge as studied by Davis *et al.* (1983) and Dahlen (1984). They define a critical taper at which failure could occur anywhere within the wedge. For larger taper than the critical, the whole décollement is activated. For a smaller taper, the deformation takes place at the back of the wedge. Several authors have predicted the position of a first thrust by introducing bulk cohesion (Yin, 1993) or elasticity (McTigue and Mei, 1981), concentrating on the onset of faulting. For the predic-

tion of thrusting sequences, two different approaches are generally adopted. The first is based on minimum work principle (Hardy *et al.*, 1998; Masek and Duncan, 1998) which often requires predetermination of the fault positions. The second is based on the finite-element method (Erickson and Jamison, 1995; Erickson *et al.*, 2001; Buitter *et al.*, 2006) which provides mechanical solution despite various computational difficulties and the need for intensive computational power. Our procedure, the maximum rock strength theorem (Maillot and Leroy, 2006) has its root in the external approach of limit analysis (Salençon, 1974) and belongs to the first class of approaches with the added benefit of a mathematical justification.

The results of sandbox experiments are presented to validate the proposed procedures. The aim is to determine probability distributions for the friction parameters by

an inverse analysis. Prior to the inversion, a statistical model of the experimental uncertainties is build.

### Prediction of thrusting sequence

#### *The Maximum rock strength theorem*

The method consists of determining the three stages of the life of a thrust: 1) the onset of thrusting along its ramp, 2) the evolution with the construction of a relief, and 3) the cessation because of the onset of another thrust.

The strategy developed by Maillot and Leroy (2006) for a kink fold and applied to an accretionary wedge by Cubas *et al.* (2008) follows the kinematics defined by geologists for the geometry of the thrust fold. Rigid regions (the backstop, the hanging wall and the toe) are separated by velocity discontinuities (décollement, ramp and backthrust) along which work is dissipated according to the Coulomb criterion. Degrees of freedom (such as the location and the dip of the ramps and the backthrusts) are then introduced in this geometry and optimized according to the maximum rock strength. This optimization leads to a least upper bound in the tectonic force. At each step of the shortening, this bound is compared with the one required for the onset of a new thrusting event. The development of the thrust or its cessation because of the initiation of another thrust is decided by selecting

the event which leads to the least upper bound. This strategy applied at each step of shortening leads to the prediction of the sequence of thrusting presented in figure 1, ended with the appearance of the first out-of-sequence thrust.

#### *Results*

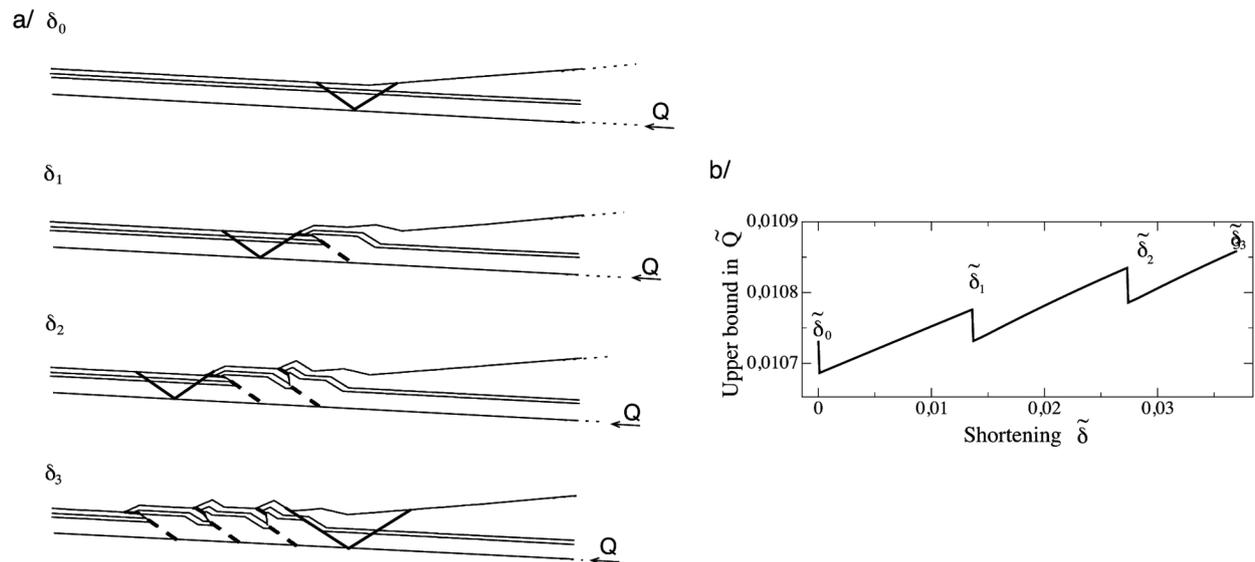
The procedure has been validated by proving that the critical taper defined by Dahlen (1984) is properly captured (Cubas *et al.*, 2008). The study of normal sequences reveals that weakening of the ramp (accounted for by changing its friction angle from an initial to a smaller final value) is necessary to predict a finite life span to each thrust.

For supercritical conditions (taper above critical), we observe how parameters control both the number and the lifetime of thrusts. For larger ramp weakening, the life span of a thrust increases together with the topography (Fig. 2a). In contrast, decreasing the décollement friction increases the number of thrusts in the sequence yielding to milder relief (Fig. 2b).

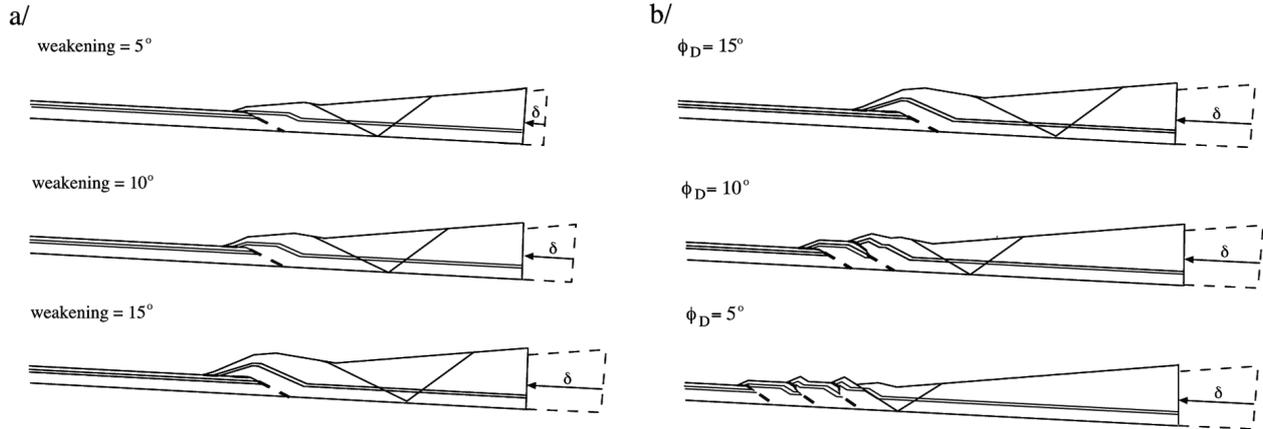
### Sandbox experiments

#### *Experimental set-up*

The experiment consists in sliding a stable sand wedge with a toe of 9 mm (Fig. 3a). The surface slope



**Figure 1.** (a) Four different stages of a predicted thrusting sequence ended with the first out-of-sequence. A tectonic force  $Q$  is applied from right to left. The two thin central lines serve as passive markers. The bold solid line represent the active ramp and backthrust, and the dashed line the previous thrusts. Parameters: surface slope:  $4.5^\circ$ , décollement dip:  $3^\circ$ , bulk friction:  $30^\circ$ , décollement friction:  $5^\circ$ , weakening on the ramp:  $15^\circ$ , (b) least upper bound  $Q$  normalized vs. shortening  $\delta$  normalized.



**Figure 2.** (a) Final stages for different values of weakening, (b) décollement friction  $\Phi_D$ . Parameters: surface slope:  $4.5^\circ$ , décollement dip:  $3^\circ$ , bulk friction:  $30^\circ$ , décollement friction:  $15^\circ$  in (a), weakening on the ramp:  $15^\circ$  in (b).  $\delta$  = shortening.

of the wedge is  $6^\circ$ , with a maximum thickness at the contact of the back wall of 17 mm. A shortening of 30 mm is applied on the wedge, at a rate of  $0.52 \text{ mm s}^{-1}$  with an electric motor, producing one or two ramps with their back-thrusts.

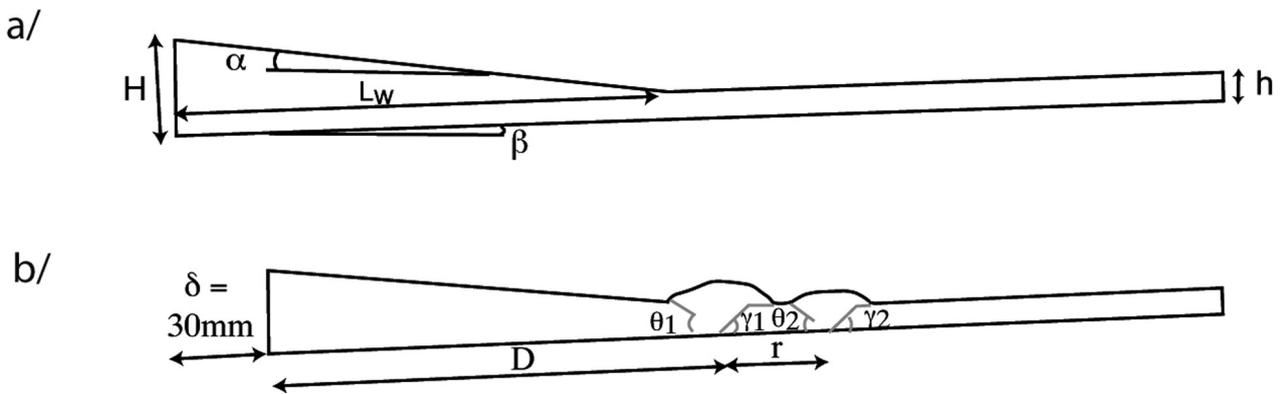
As the theory provides a 2D prediction, the first step was to select cross-sections which were not affected by side-walls effects. For this purpose, we used a specific box, of 40 cm length and 28 cm width, permitting a reversible motion of side walls (Fig. 4a). A sand distributor, providing a homogeneous and reproducible density over the whole box, was used. Strain gages were placed at the front and the back of the box in order to measure the applied force to be compared to the predictions. Each experiment was repeated five times (to evaluate the reproducibility, and to multiply the number of observables) with lateral walls accom-

panying the sliding (configuration A), and five times with fixed lateral walls (configuration B). By superimposing the results of all experiments, we determined a central zone devoid of side-walls effects (Fig. 4b).

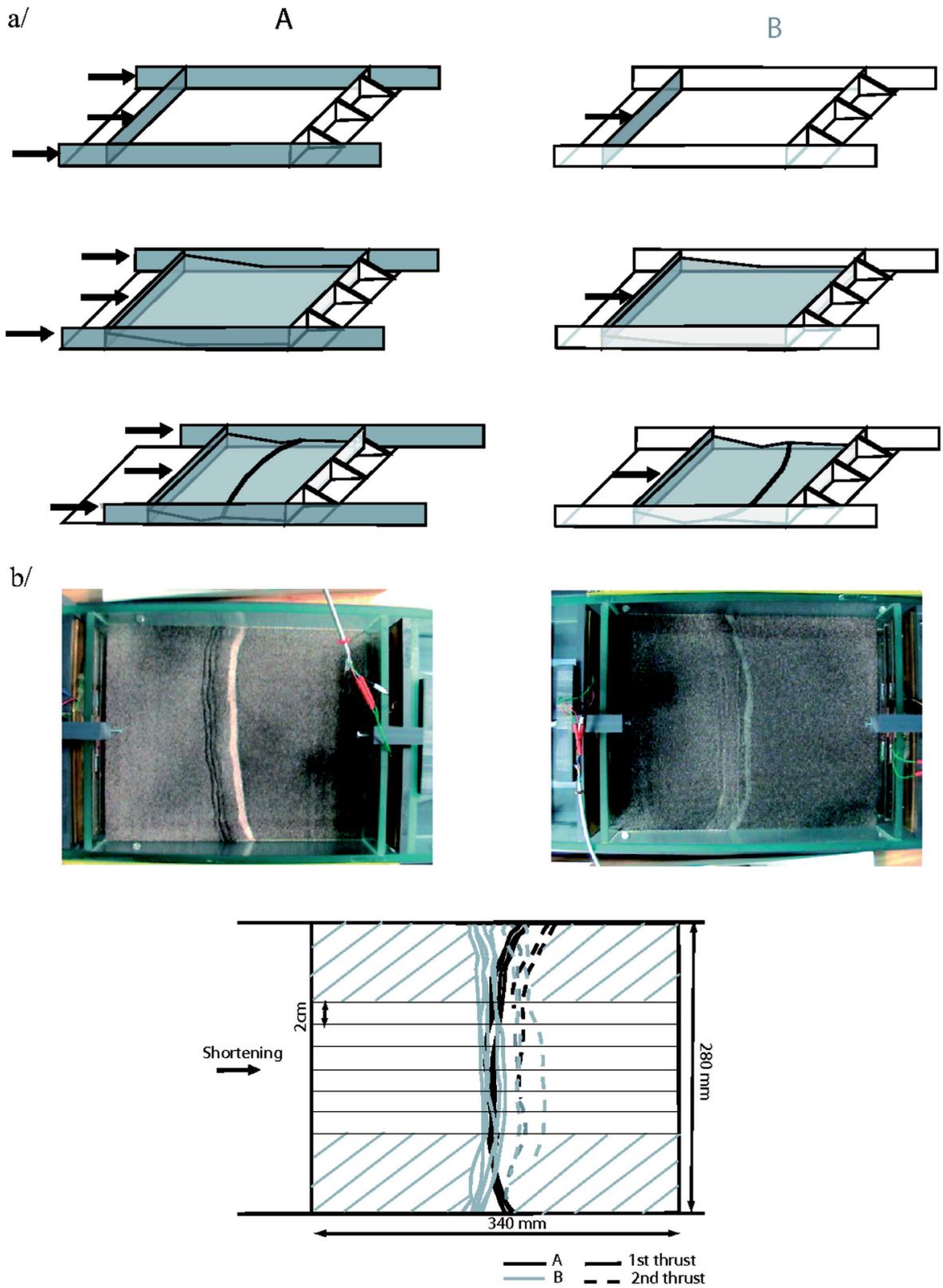
In that zone, we measured on several cross-sections what we defined as the observables: the location, dip, and life span of the two first thrusts developing at the front of the wedge (Fig. 3b). The fault strength will be deduced from these observables and their statistical distributions, by an inverse analysis.

*Statistical model*

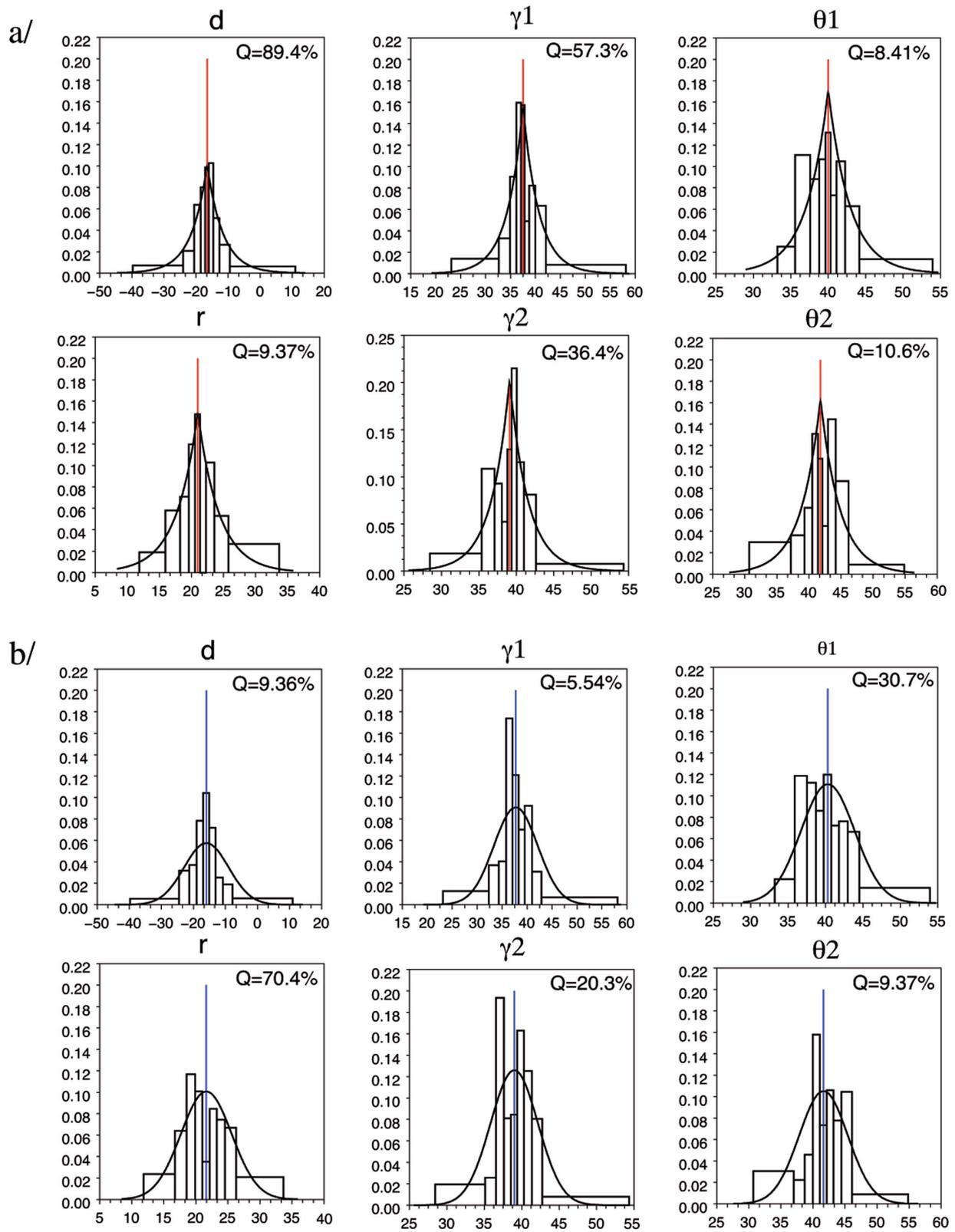
Before defining the statistical model corresponding to each observable, the reproducibility and the independence of measurements on each cross-section are established. We show that variations of the observ-



**Figure 3.** (a) Experimental set-up ( $\alpha = 6^\circ$ ,  $H = 17 \text{ mm}$ ,  $h = 9 \text{ mm}$ ,  $L_w = 162 \text{ mm}$ ), (b) observables ( $d = D - L_w$  location of the 1<sup>st</sup> thrust related to the front of the wedge,  $\gamma_1 = 1^{\text{st}}$  ramp dip,  $\theta_1 = 1^{\text{st}}$  backthrust dip,  $r =$  distance between 1<sup>st</sup> and 2<sup>nd</sup> thrust,  $\gamma_2 = 2^{\text{nd}}$  ramp dip,  $\theta_2 = 2^{\text{nd}}$  backthrust dip) and shortening  $\delta$ .



**Figure 4.** (a) Two possible configurations of the sandbox, A: side-walls accompanying the sliding, B: fixed lateral walls, (b) top view of final deformation in configuration A and B and superposition of all experiments, determination of a zone devoid of side-walls effect and localization of cross-sections.



**Figure 5.** Observables histograms and probabilities (Q) for (a) Gaussian distribution with the mean, (b) Laplacian distribution with the median. (d : location 1st ramp according to the front of the wedge,  $\gamma_1$ : 1<sup>st</sup> ramp dip ,  $\theta_1$ : 1<sup>st</sup> backthrust dip, r: distance between 1<sup>st</sup> and 2<sup>nd</sup> thrust,  $\gamma_2$ : 2<sup>nd</sup> ramp dip,  $\theta_2$ : 2<sup>nd</sup> backthrust dip).

ables are similar across experiments, and that a 2 cm distance between each cross-section guarantees the independancy of observables. Each cross-section can thus be considered as an independent experiment. This useful assumption permits to reduce the number of experiments necessary to build statistics of uncertainties. After proving that the observables are independent from each other, we calculate the statistical distribution (either gaussian or laplacian) followed by each observable uncertainties, according to the  $\chi^2$  test (Fig. 5). These distributions are then extrapolated to other experiments with differing relief and basal slopes. This provides a probabilistic data set for inversion purposes, avoiding the need to repeat them.

The distribution of distances (location of first ramp related to the front of the wedge, and distance between first and second ramp) appears to be more diffuse than that of the dips (of ramps and back-thrusts). It appears that dips are strongly controlled by the sand rheology, whereas distances are more sensitive to the construction of the wedge, dependant of experimenter interventions.

## Conclusion

We demonstrate that the maximum strength theorem provides predictions for the location of the active thrust, dips of ramp and backthrust, but also the amount of shortening accommodated during its life span. The proposed procedure has two advantages: 1) a theoretically rigorous upper bound on the tectonic force, 2) faults are not pre-

defined but activated at the most favorable location as the consequence of optimization. Analogue experiments are carried out to validate the procedure. The analogue benchmark conducted by Schreurs *et al.* (2006) highlights the necessity to construct a statistical model of the observables prior to the inverse analysis. Once the misfit between the real and predicted data is calculated, as a function of the type of data uncertainty (gaussian norm or laplacian norm), we determine probability distributions of the friction parameters. These will be compared to independant measurements.

The future step of this line of work is to explore various modes of folding and to select and optimize the one leading to the least upper bound. To do this, we are presently studying the Agrio fold-and-thrust belts of the Neuquén region in Argentina. The structure (thinned or thicked-skin) of the fold-and-thrust belts is a matter of debate (Kozłowski *et al.*, 1996; Manceda and Figueroa, 1995; Zapata *et al.*, 1999). We propose to introduce in the next procedures several levels of décollement and to allow propagation landward. We expect to raise mechanical arguments permitting to choose between these two different kinematics.

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## References

- BUITER, S. J. H., BABEYKO, A. Y., ELLIS, S., GERYA, T. V., KAUS, B. J. P., KELLNER, A., SCHREURS, G. and YAMADA, Y. (2006): The numerical sandbox: comparison of model results for shortening and an extension experiment. In: S. J. H. BUITER and G. SCHREURS (eds): *Analogue and Numerical Modeling of Crustal-Scale Processes. Geol. Soc. London Spec. Publ.*, 253: 29-64.
- CUBAS, N., LEROY, Y. M. and MAILLOT, B. (2008): Prediction of thrusting sequences in accretionary wedges. *J. Geophys. Res.*, 113: B12412, doi:10.1029/2008JB005717.
- DAHLEN, F. A. (1984): Noncohesive critical Coulomb wedges: an exact solution. *J. Geophys. Res.*, 89, 12: 10125-10133.
- DAVIS, D., SUPPE, J. and DAHLEN, F. A. (1983): Mechanics of Fold-and-Thrust Belts and Accretionary Wedges. *J. Geophys. Res.*, 88, 2: 1153-1172.
- ERICKSON, S. G. and JAMISON, W. R. (1995): Viscous-plastic finite-element models of fault-bend folds. *J. Struct. Geol.*, 17: 561-573.
- ERICKSON, S. G., STRAYER, L. M. and SUPPE, J. (2001): Initiation and reactivation of faults during movement over thrust-fault ramp: numerical mechanical models. *J. Struct. Geol.*, 23: 11-23.
- HARDY, S., DUNCAN, C., MASEK, J. and BROWN, D. (1998): Minimum work, fault activity and the growth of critical wedges in fold and thrust belts. *Basin Res.*, 10: 365-373.
- KOZŁOWSKI, E. E., CRUZ, C. E. and SYLWAN, C. A. (1996): Geología estructural de la zona de Chos Malal, cuenca Neuquina, Argentina. *XIII Congreso Geológico Argentino y III Congreso de Exploración de Hidrocarburos, Buenos Aires, Acta*, 1: 15-26.
- MAILLOT, B. and LEROY, Y. M. (2006): Kink-fold onset and development based on the maximum strength theorem. *J. Mech. Phys. Solids*, 54: 2030-2059.
- MANCEDA, R. and FIGUEROA, D. (1995): Inversion of the Mesozoic Neuquén rift in the Malargüe fold and thrust belt, Mendoza, Argentina, in *Petroleum Basins of South America. AAPG Mem.*, 62: 369-382.

- MASEK, J. G. and DUNCAN, C. C. (1998): Minimum-work mountain building. *J. Geophys. Res.*, 103, 1: 907-917.
- MCTIGUE, D. F. and MEI, C. C. (1981): Gravity-induced stresses near topography of small slope. *J. Geophys. Res.*, 86, 10: 9268-9278.
- SALENÇON, J. (1974): *Théorie de la plasticité pour les applications à la mécanique des sols*, Ed. Eyrolles, Paris, 178 pp.
- SCHREURS, G., BUITER, S. J. H., BOUTELIER, D., CORTI, G., COSTA, E., CRUDEN, A. R., DANIEL, J. M., HOTH, S., KOYI, H. A., KUKOWSKI, N., LOHRMANN, J., RAVAGLIA, A., RAVAGLIA, R. W., WITHJACK, M. O., YAMADA, Y., CAVOZZI, C., DELVENTISETTE, C., BRADY, J. A. E., HOFFMANN-ROTHER, A., MENGUS, J. M., MONTANARI, D. and NILFOROUSHAN, F. (2006): Analogue benchmarks of shortening and extension experiments, in Analogue and numerical modelling of crustal-scale processes. *Geol. Soc. London Spec. Publ.*, 253: 1-27.
- YIN, A. (1993): Mechanics of Wedge-Shaped Fault blocks 1. An elastic solution for compressional wedges. *J. Geophys. Res.*, 98, 8: 14245-14256.
- ZAPATA, T., BRISSON, I. and DZELALIJA, F. (1999): La estructura de la faja plegada y corrida andina en relación con el control del basamento de la Cuenca Neuquina. *B. Info. Pet.*, 60: 112-121.